Organizational Capacity and Project Dynamics*

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Abstract

This paper provides a dynamic theory of the effects of organizational capacity on public policy. Consistent with prevailing accounts, a bureaucratic organization with higher capacity, i.e., a better ability to get things done, is more likely to deliver projects in a timely, predictable, or efficient fashion. However, capacity also interacts with political institutions to produce far-reaching implications for the size and distribution of public projects. Capacity-induced delays and institutional porousness can allow future political opponents to revise projects in their favor. In response, politicians design projects to avoid revisions, for example by equalizing distributive benefits, or by over-scaling projects. We show that higher organizational capacity can increase project size, inequalities in the distribution of project benefits, and delays. The capacity levels that minimize social benefits increases with the extent of institutional constraints, suggesting that political systems with high capacity and high institutional constraints are especially vulnerable to inefficient projects.

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1 Introduction

It is now a truism that organizations are crucial for the outcome of government policies in modern society. Election candidates can make platform promises and legislators can pass laws, but a massive bureaucratic machinery is needed to translate statutes into on-the-ground results.[1] Capturing organizational performance is obviously a formidable task, but practitioners and scholars have increasingly coalesced around the concept of *organizational capacity* as a central determinant. Bodies as varied as the UNDP, USAID, OECD (2011), and the European Centre for Development Policy Management (2011) identify organizational capacity as a key development objective, and scholarly mentions of the term have increased sharply since the 1990s.[2]

The appeal of organizational capacity is clear. Higher capacity — loosely speaking, a better ability to "get things done" — should produce policy outputs that are more timely, more efficient, or of higher quality. Consistent with this perspective, a wide variety of studies have shown that organizations that are under-resourced, under-paid, or prone to political interference produce worse results (e.g., Derthick, 1990; Rauch and Evans, 2000; Gorodnichenko and Peter, 2007; Propper and Van Reenen, 2010). Yet in many political settings, the implications of capacity are less obvious. To take a simple example, suppose that a political system gives broad legal standing to actors who have environmental objections to a construction project. In this setting, a high-capacity bureaucracy might actually encourage litigation and its attendant delays, since victorious litigants can be confident that their proposals will be implemented quickly.

This paper develops a dynamic theory of policy-making that jointly considers organiza-

[1]The Organisation for Economic Co-operation and Development estimates that as of 2019, government entities accounted for an average of 18% of member country employment (OECD, 2021).

[2]See "Capacity Development: A UNDP Primer," USAID's "Measuring Organizational Capacity." As of August 2022, Google Scholar returned about 4,880 results for "organizational capacity" between 1990 and 1999, 16,000 between 2000 and 2009, 23,400 between 2010 and 2019, and 14,600 since 2020.

tional capacity and its political and institutional context. Its main objective is to show how these features combine to affect the planning and execution of public policies, in terms of scale, distribution of benefits, and delays. While many elements of our model are standard, the principal hurdle in any such effort is the lack of consensus about how to characterize organizational capacity. A predominant approach in empirical research is to treat capacity as an input into organizational production functions. Such inputs include information (Lee and Zhang, 2017) and perhaps most prominently, human capital (Brown et al., 2009; Dal Bó et al., 2013; Acemoglu et al., 2015; Bolton et al., 2016). Theoretical efforts have thus far adopted widely divergent perspectives on how to incorporate the concept into standard political economy frameworks, ranging from the variance of policy outcomes (Huber and McCarty, 2004), policy valence (Ting, 2011), to agency cost structures (Foarta, 2022).

Our conceptualization of capacity blends many of the insights of existing approaches. Its basis is a discrete Markov process representation of policy projects. Completing a project requires traversing a sequence of bureaucratic stages. Criminal prosecutions might start with an investigative stage and end with a trial. As DiIulio (2014) observes, US federal contracting also consists of a series of distinct stages:

> [T]he federal contracting process has three separate but related parts: (1) planning (how federal agencies decide what and how much to contract for, when they need given goods or services to be delivered, and what terms and conditions are they subject to); (2) awarding (the background market research, the communications and outreach to prospective contractors, the budgetary criteria, and the precise procedures for awarding competitive bids or making noncompetitive selections); and (3) overseeing (everything from routine reporting requirements to financial audits, field inspections, public comments, and impact studies).

Capacity is the probability p of progressing from each stage to the next. With probability

$1-p$, the project does not progress. Benefits are realized upon completion, but each period before completion imposes costs that are increasing in the project's scale. Thus in the absence of renegotiation or political interference, an agency with higher capacity — due to better personnel or technology — reduces costs and variability in delivery times.

The model embeds this process in an institutional environment that gives access to political opponents. At the inception of a project, a politician from one group chooses its scale and an initial distribution of benefits between its group and an opposing group. This distribution may represent a siting choice, or the selection of contractors. After the project begins, groups randomly receive opportunities to attempt to revise the project. Depending on the political system, these opportunities can arise from various sources, for example the election of new politicians or the mobilization of NIMBY groups. Attempting a revision delays project completion by automatically pausing progress, and the revision itself succeeds with some probability that corresponds to the openness of the institutional environment to outside intervention. This openness reflects factors such as contracting regulations, the judicial system, or administrative procedures such as the US National Environmental Policy Act (NEPA) review process. A successful revision changes the project's distribution of payoffs to favor the revising party. The original project designer must then take the possibility of strategic revisions into account in choosing the project's scale and payoff split; in particular, one liability of low capacity is the increased opportunity for political intervention during the course of project execution.

A principal attraction of this formulation is its correspondence to the operational realities of implementing many public policies. A good example is the process of constructing large infrastructure projects in the US.[3] The federal government's main mechanism for supporting significant public transportation projects is the Federal Transit Administration (FTA)

[3]The Federal Infrastructure Projects Permitting Dashboard tracks the progress of federally-funded infrastructure projects across major permitting requirements. The Center for an Urban Future provides an overview of the key phases and sources of delay for capital construction projects in New York City.

Capital Investment Grants (CIG) program. CIG administers over $2 billion a year through a competitive grant process, whereby state or local transit agencies propose cost-sharing collaborations with the FTA. Applications must traverse two stages of FTA review before construction can begin. The first, "Project Development," requires a completed NEPA review, approval by local authorities, and secured commitments for at least 30% of non-federal funds. The second, "Engineering," finalizes funding sources and design details, including geotechnical and safety hazard reports. Each phase can be a lengthy undertaking, thus exposing projects to both lawsuits and political turnover.

We find that the interaction between capacity and the institutional environment has significant implications for public projects. Consider starting from a benchmark in which the opposition group never has an opportunity to attempt a revision. In this case, higher capacity has the straightforward effects of reducing completion time and costs, thereby increasing project scale. The initiating politician furthermore awards herself the entire benefit of the project. If the opposition group is given the opportunity to attempt a revision, then the threat of the project being revised has two possible effects. First, it encourages the project initator to design a larger project. The high running costs of such a project deter revisions due to the prohibitive escalations in total costs. Second, it encourages more equal payoff divisions, as these reduce the gains from revisions. These detterence effects matter only to politicians who are relatively likely to face future revision attempts: the side that is unlikely to have revision opportunities will typically not attempt revisions, since their revisions are likely to be reversed. Thus, a politically favored initial politician can achieve her benchmark ideal policy, but an unfavored politician is more likely to distort the size and distribution of its projects in order to avoid revisions. When capacity is very low, unfavored politicians choose more egalitarian distributions and (to compensate for the reduced project gains) under-scaled projects. As capacity increases, they claim an increasing share of project benefits and switch to over-scaling. In all cases, high capacity results in winner-take-all allocations.

These results feature no politically-induced delays in equilibrium, but they assume that politicians can freely choose any project scale. Also, they assume that scale increases do not augment running costs so much as to make the project altogether undesirable. In practice, both of these concerns are may be present. Scales are often constrained by budgets or physical limitations. Even when physically possible, increasing scale may lead to rapidly raising costs (if the cost function is very elastic). Such conditions could make an over-scaling strategy unattainable. Modest scales and high capacity imply low running costs, and thereby encourage revisions. The surprising implication is that *higher capacity produces greater obstruction* and delay.

The adjustments that project designers make to avoid revisions have important implications for social welfare. Under-scaled and benchmark projects generally provide greater benefits than costs to society, but the agents can collectively do much worse in expectation when projects are over-scaled. In addition to depending on capacity, over-scaling depends on its "match" with the ease of outside intervention. A political system with high capacity and openness is most prone to over-scaling, while systems with mismatched capacity and openness will be less so.

We finally explore a variant of the model with a more complex project that requires two phases. Here, scales are chosen independently the initial incumbent politician in each phase and the output of the first phase is an "investment" that reduces costs for the final project in the second. The main result is that the first project initiator may now invest nothing and effectively cancel a project if it worries about possible over-scaling by the opponent. Thus, the prospect of setting project parameters mid-stream can force politicians to internalize welfare consequences to some degree.

Related Literature. The main contribution of this paper is its formalization of organizational capacity as part of a dynamic political process. The execution of policy in our model generates measurable outcomes such as the size, timing, cost, and distributive dimensions of public projects. Several important lines of theoretical work have used related notions of capacity to explore different policy questions. Perhaps most prominently, a recent literature on "state capacity" addresses the ability of the state to achieve macro-objectives such as tax collection and law enforcement (Besley and Persson, 2009; Johnson and Koyama, 2017). One emphasis of this work is the creation of capacity in the shadow of political transitions. By contrast, we address policy-making at the organizational level, taking capacity as given. The granular focus on organizations can be useful because, as many observers have noted, organizational capabilities can vary greatly within a country (Carpenter, 2001).

A series of models by Huber and McCarty (2004, 2006) situates bureaucratic capacity in an explicit institutional setting. They examine the relationship between a legislative principal and a bureaucratic agent, and represent capacity as the variance of possible outcomes following a bureaucratic policy choice. The outcome space in these models is ideological, and the primary outputs include delegation, compliance, and whether legislation is possible. Other institutional theories that model capacity as costs include Foarta (2022) and Turner (2020), who analyze a dynamic electoral setting and policy-making in a separation of powers system, respectively. Aside from a different set of outcomes, one contribution of the present paper is a formalization of organizational capacity that can generate both variance and costs.

Finally, a now extensive set of theoretical models addresses the dynamics of long-term policies (e.g., Baron, 1996; Battaglini et al., 2012). Similarly, a growing contracting literature studies the optimal provision of incentives in dynamic environments with multiple stage projects (e.g., Toxvaerd, 2006; Green and Taylor, 2016; Feng et al., 2021). Yet, there is little theoretical work on the political economy of large multistage public investments.[4]

[4]Focusing on transportation projects specifically, Glaeser and Ponzetto (2018) develop a simple model of project scale, focusing on voter inattention as the driver for politicians to propose very large projects: increased voter attention to local negative externalities leads to reductions in project scale, and is consistent with evidence of positive correlation between voter education and highway costs.

2 Examples and Motivating Cases

The parameters and mechanisms of our model map into commonly observed features of bureaucracies and public projects. In this section we provide examples of how some of the main components of the model have appeared in the implementation of public policies.

Capacity and Delays. Our p parameter captures an organization's ability to solve discrete problems. This parameter perhaps corresponds mostly closely with prevailing empirical notions of capacity, which often emphasize human capital. There are numerous examples of shortfalls in human capital reducing bureaucratic productivity. Understaffing at the US Office of Information and Regulatory Affairs has been shown to delay the issuance of federal rules, including the Biden administration's current efforts to update energy efficiency standards for lighting and appliances (Bolton et al., 2016).[5] DiIulio (2014) observes that the hiring of 3,500 highly trained acquisition personnel between 2010 and 2013 greatly improved on-time performance of contracting tasks at the US Department of Defense.

Revisions and Distribution. Even the most competent public organizations—fully staffed with well-trained, well-paid, and uncorrupt bureaucrats, and equipped with modern technology—face political scrutiny in executing their tasks. As projects become large and politically prominent, the opportunities for political intervention multiply, and especially so in decentralized institutional systems (Pressman and Wildavsky, 1984). While elections play an obvious role, academic and policy observers have also increasingly focused on non-electoral mechanisms such as NEPA or California Environmental Quality Act reviews as sources of delay, and cost inflation in US infrastructure construction (Smith et al., 1999; Brooks and

[5]See Anna Phillips, "Biden faces delays in undoing Trump's war on efficient dishwashers, dryers and lightbulbs that made him 'look orange'." *Washington Post*, January 9, 2022.

Liscow, 2021; Mehrotra et al., 2019).[6]

Altshuler and Luberoff (2003) document multiple examples of political interventions that altered the distribution of payoffs of major infrastructure projects. The design of the Boston Central Artery/Tunnel Project (better known as the "Big Dig") was adjusted numerous times in response to state and federal officials, as well as to litigation threats over environmental impacts. Modifications addressed issues such as tunnel size, public transportation, air quality, land takings, parking, and interchange design, and contributed to raising the tunnel's estimated cost from \$3.1 billion to \$5.2 billion between 1987 and 1991. In 1970s Atlanta, mayor Maynard Jackson successfully moved the construction a new airport to a site closer to his political base in the south of the city, and also won set-asides for minority-owned firms.

Project Design. Altshuler and Luberoff (2003) observe that mid-20th century American infrastructure projects faced unprecedented political challenges, often from mobilized interest groups. In response, late-20th century projects both spread payoffs across a wider base of constituents and expanded in size. While they do not specifically invoke the equilibrium logic of our model, these tactics correspond to the ways in which its project designers avoid costly revisions.

3 Model

Consider an environment with infinite, discrete time, $t = 0, 1, 2....$ There are two agents, A and B, representing two distinct political constituencies. Agent A is in control of policy at time 0, and may be thought of as a politician in power at that time. Agent B is the opposition, either another politician or an outside interest group opposed to A. Agent A

[6]The NYU Transit Costs Project provides a useful overview of the factors that drive transportation costs in modern infrastructure projects.

initiates a long-term project at time 0. Once initiated, the project is run by a bureaucratic agency, and it must go through several stages before reaching completion. The bureaucracy is a non-strategic player. It works at its maximum capacity to move the project through the required stages, where each stages lasts at least a period. At the end of each period, a transition in control may occur, and the new agent in may attempt to change aspects of the incomplete project. The game ends when the project is completed.

The Project. A public project delivers value $v > 0$ per unit produced. It has two main characteristics which are chosen at its initial conception: (1) the scale s, i.e., the number of units that are produced, and (2) the split of the benefits between the two agents: the fraction $w \geq 0.5$ of benefits that goes to one agent, and the fraction $1 - w$ that goes to the other. The project inequality is therefore captured by $\Delta = 2w - 1 \in [0, 1]$, where $\Delta = 1$ is maximum inequality and $\Delta = 0$ is the equal division of benefits. The project starts in stage d (the development stage), and it must reach stage e (execution) in order to be completed.

The project delivers its benefits only upon completion. Progression from one stage to the next depends on the organizational capacity of the bureaucracy. The higher is the capacity of the bureaucracy, the faster it can overcome the hurdles needed in order to move the project forward. We parameterize the capacity of the bureaucracy by p, the probability with which the project moves from stage d to stage e in any given period. With probability $1 - p$, the project does not progress that period. Every period spent in stage d costs each agent $c(s)$, where we assume the following about the cost function:

Assumption 1 *The cost function satisfies $c'(s) > 0$, $c''(s) > 0$, $c(0) = 0$, and has elasticity*

$$\varepsilon(s) \equiv \frac{c'(s) \cdot s}{c(s)} \geq 1.$$

The project cost $c(s)$ is incurred by each agent every period while the project is running.

This captures in reduced from the use of general tax revenue for public projects, regardless of the final division of benefits from these projects. The cost is increasing and convex in s. Its elasticity with respect to s, $\varepsilon(s)$ is larger than 1 to ensure that $\frac{c(s)}{s}$ is increasing in s, i.e., larger projects are relatively costlier.

Transitions of Control and Revisions. At the beginning of period 0, agent A chooses the scale of s and fraction w (implicitly the benefit inequality Δ), where agent A receives fraction w of $v \cdot s$ and agent B receives fraction $1 - w$. We refer to this distribution of benefits as the project of type Δ^A, as the benefit inequality favors agent A, and denote by Δ^B the project where fraction w of the benefit goes to agent B. At the end of each period t, control over policy may change. With probability r, agent A has control next period. With probability $1 - r$, agent B gets control. The agent in control may then choose to trigger a project revision. Once triggered, a revision freezes the project for the current period, so that it cannot advance to the next stage. With probability q, the revision is successful and it changes the project type, so that the agent in control receive fraction w of the value. There is no additional cost of triggering a review. The probability of a successful revision, q, captures in reduced form the insitutional or legal ease to win appeals or amend ongoing projects.

Payoffs. A project of type Δ_i completed after $\mathbb{T}$ periods has payoff to agent $i \in \{A, B\}$

$$w \cdot v \cdot s - \mathbb{T} \cdot c(s). \tag{1}$$

Timing. To summarize, the timing is as follows. In period 0, agent A, the politician in control, starts a project and chooses its scale s and payoff inequality Δ. In each period $t \geq 1$, while the project is in stage d:

1. With probability r, agent A has control over the project; with probability $1 - r$, agent B has control.
2. The agent in control chooses whether to trigger a revision.
3. If a revision is triggered, it succeeds with probability q, and the project type switches from Δ_i to Δ_j, where $i \neq j$; with probability $1 - q$ the revision fails, the project type does not change, and the project remains in stage d for the period.
4. If a revision is not triggered, then the project moves to stage e with probability p; with probability $1 - p$ it remains in stage d for the period.
5. Each agent pays the project operating cost $c(s)$ for the period.

Once the project reaches stage e, its benefits are realized given the current project type. There is no discounting between periods.

Equilibrium Concept. We derive the Markov Perfect Equilibria of this game with state variables for periods $t \geq 1$ being the current project stage $S_t \in \{d, e\}$, the agent $P_t \in \{A, B\}$ who has control over the project, and the project type $\Delta_i \in \{\Delta^A, \Delta^B\}$. In period 0, the state variable is $P_0 = A$. Each period $t \geq 1$, agent P_t in control chooses a probability of revision $\sigma^{P_t}(\Delta_i) \in [0, 1]$ to maximize her expected utility. In period 0, agent P_0 chooses s and w to maximize her expected utility.

We note that any strategy in which an incumbent i revises a project of type i is weakly dominated. Thus, $\sigma^i(\Delta_i) = 0$, and simplify notation by denoting $\sigma^i \equiv \sigma^i(\Delta_j)$, for $i \neq j$.

4 Benchmarks

Our model captures two key aspects of long-term public projects. First, the project initiator has the freedom to establish key project characteristics, as its size and distribution of benefits across constituencies. Second, opposition politicians or groups have opportunities to modify the project. To understand what these aspects mean for the initial setup and the dynamics of projects, we first analyze two benchmark cases, where these aspects are fully or partially removed.

4.1 The Social Planner Solution

We start with the case in which the project is started and managed by a social planner who maximizes the average utility of the two agents. Under the social planner benchmark, there is no inequality in payoffs ($\Delta = 0$) and no transitions of control. The social planner chooses s to solve

$$\max_{s \in [0, s^{\max}]} v \cdot s - \mathbb{T}(p) \cdot 2 \cdot c(s), \tag{2}$$

where $\mathbb{T}(p)$ is the expected time until the project reaches stage e given bureaucratic capacity p. That implies $\mathbb{T}(p) = \frac{1}{p}$. The socially optimal size of the project is then given by

$$c'(s^{SP}) = \frac{vp}{2}. \tag{3}$$

4.2 No Transitions of Control

The second benchmark is the one where there are no transitions ($r = 1$), so that agent A starts in control in period 0 and remains in control until the project reaches execution. This implies that agent A never revises the project on the equilibrium path, and she chooses $w \in [0, 1]$ and $s \in [0, s^{\max}]$ to maximize

$$\max_{s,w} w \cdot v \cdot s - \mathbb{T}(p) \cdot c(s), \tag{4}$$

where, as above, $\mathbb{T}(p) = \frac{1}{p}$. Agent A will choose a project that delivers all the benefits to herself ($\Delta = 1$) and scale given by

$$c'(s^{NT}) = vp. \tag{5}$$

Agent A does not internalize the cost borne by Agent B, and therefore increases the scale compared to the social planner.

5 Public Projects under Transitions of Control

We now move to analyzing the full model, highlighting the role that bureaucratic capacity and transitions of control play in determining the project scale and distribution of benefits. Turnover in control opens the path of project revisions, and therefore to delays in project progress. This translates to longer time to project completion and higher running costs. The key to our analysis will be to understand if and when revisions occur and what that implies for the initial characteristics of projects. On the one hand, the expectation of higher running costs due to revisions should decrease the initial scale chosen in period 0 and increase benefit inequality Δ, as each revision can swing the project in one's favor. On the other hand, increasing the initial scale or reducing Δ could be used strategically to discourage revisions.

To solve for the equilibrium project scale, distribution of benefits, and the path of revisions, we break up the problem into two main steps. First, for a given scale s and payoff inequality Δ, we find the optimal revision strategy for each agent in period $t \geq 1$. Second, we find the s and Δchosen by agent A at time 0 given the expected continuation play.

5.1 The Revision Response

Assume a fixed s and w. In each period $t \geq 1$, the project's evolution into the next period can be represented as a Markov Process with six states, given the possible combination of stage, controlling agent, and project type. The probability of the project moving from its current state to any of the possible states depends on the probability of a transition,r, the bureaucracy's capacity to move the project to the execution stage,p, and the revision probabilities σ^A and σ^B. A project at stage e is in an absorbing state, with payoffs given its type, Δ^A or Δ^B. The Markov process is represented in matrix form in Figure 1.

Starting in a state (d, i, Δ_k) with agent i in control and project type Δ_k, the Markov transition probabilities imply an expected probability of reaching stage e with project type Δ_ℓ of $\mathbb{P}(e, \Delta_\ell | d, i, \Delta_k)$ and an expected number of periods needed to reach stage (e, Δ_ℓ) of $\mathbb{T}(e, \Delta_\ell | d, i, \Delta_k)$. We can use these objects to compute the expected utility for each agent, starting from any project stage, for revision strategies σ^A and σ^B. For agent A, the expected utility given controlling agent $i \in \{A, B\}$ and current project type Δ_k is:

$$U^A(i, \Delta_k) = \mathbb{P}(e, \Delta^A) \cdot w \cdot v \cdot s + \mathbb{P}(e, \Delta^B) \cdot (1 - w) \cdot v \cdot s \\ - [\mathbb{P}(e, \Delta^A) \cdot \mathbb{T}(e, \Delta^A) + \mathbb{P}(e, \Delta^B) \cdot \mathbb{T}(e, \Delta^B)] \cdot c(s). \quad (6)$$

For agent B, the only difference is in the payoffs at each terminal state: fraction $1 - w$ of $v \cdot s$ at (e, Δ^A) and fraction w at (e, Δ^B).

Given revision probability σ^i, where $i \in \{A, B\}$, agent $j \neq i$ prefers to revise if her expected utility from doing so is higher than the expected utility from continuing with the current project type. Revisions will be less likely as the scale of the project increases, and running costs increase with it. In the Appendix, we show formally that there exist cutoffs $s_1 \leq s_2 \leq s_3$ that determine the revision strategies:

Figure 1: Project Evolution as a Markov Process

	d, A, Δ^A	d, A, Δ^B	d, B, Δ^A	d, A, Δ^B	e, Δ^A	e, Δ^B
d, A, Δ_A	$(1-p)r$	0	$(1-r)(1-p)$	0	p	0
d, B, Δ^B	$qr\sigma^A$	$(1-p)r(1-\sigma^A)$ $+(1-q)r\sigma^A$	$q(1-r)\sigma^A$	$(1-p)(1-r)(1-\sigma^A)$ $+(1-q)(1-r)\sigma^A$	0	$p(1-\sigma^A)$
d, B, Δ^A	$(1-p)r(1-\sigma^B)$ $+(1-q)r\sigma^B$	$qr\sigma^B$	$(1-p)(1-r)(1-\sigma^B)$ $+(1-q)(1-r)\sigma^B$	$q(1-r)\sigma^B$	$p(1-\sigma^B)$	0
d, B, Δ^B	0	$(1-p)r$	0	$(1-p)(1-r)$	0	p
e, Δ^A	0	0	0	0	1	0
e, Δ^B	0	0	0	0	0	1

Note: Transition matrix for the project. Each state of the Markov Process is given by the project stage (d or e), controlling agent (A or B), and project type (Δ^A or Δ^B).

Lemma 1 *There exist thresholds* $\bar{s}_1, \bar{s}_2, \bar{s}_3$ *such that in any period* $t \geq 1$*, the MPE is*

- *If* $\frac{c(s)}{s} \leq \frac{c(\bar{s}_3)}{\bar{s}_3}$*, then the project is revised every time the opposition gains control:* $\sigma^A = \sigma^B = 1$*.*

- *If* $\frac{c(\bar{s}_3)}{\bar{s}_3} < s \leq \frac{c(\bar{s}_2)}{\bar{s}_2}$*, then the project is revised only by the electorally advantaged agent* $\sigma^A = 1, \sigma^B = 0$ *if* $r \geq \frac{1}{2}$*, and* $\sigma^A = 0, \sigma^B = 1$ *if* $r < \frac{1}{2}$*;*

- *If* $\frac{c(\bar{s}_2)}{\bar{s}_2} < s < \frac{c(\bar{s}_1)}{\bar{s}_1}$*, then either exactly one agent revises (*$\sigma^A = 1, \sigma^B = 0$ *or* $\sigma^A = 0, \sigma^B = 1$*) or there is a mixed strategy equilibrium with* $\sigma^A, \sigma^B \in (0, 1)$*.*

- *If* $s \geq \frac{c(\bar{s}_1)}{\bar{s}_1}$*, then the project is never revised:* $\sigma^A = \sigma^B = 0$*;*

The equilibrium regions are represented in Figure 2. As shown in the Appendix, the threshold values are given by

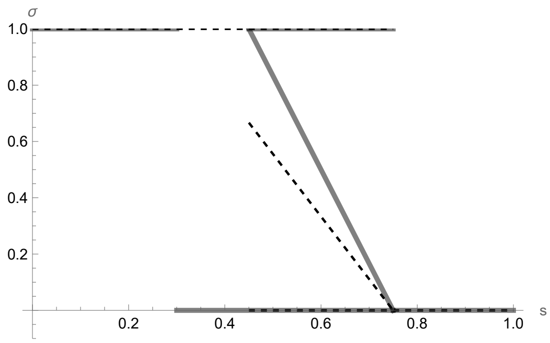
$$\frac{c(\bar{s}_1)}{\bar{s}_1} = qv\Delta, \tag{7}$$

$$\frac{c(\bar{s}_2)}{\bar{s}_2} = qv\Delta \cdot \max\left\{\frac{pr}{pr + 2q(1-r)}, \frac{p(1-r)}{p(1-r) + 2qr}\right\}, \tag{8}$$

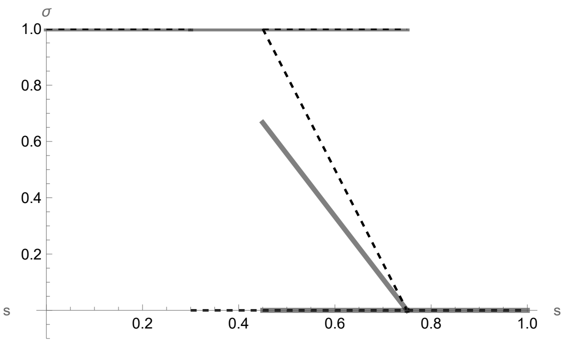
$$\frac{c(\bar{s}_3)}{\bar{s}_3} = qv\Delta \cdot \min\left\{\frac{pr}{pr + 2q(1-r)}, \frac{p(1-r)}{p(1-r) + 2qr}\right\}. \tag{9}$$

The intuition for this result comes down to the trade-off implied by a revision: a successful revision leads to a change in payoffs of $vs\Delta$; yet, it comes at the cost of project delays, with total costs proportional to $c(s)$. If s is relatively large, then any delay generated by a revision is too costly, regardless of whether the other agent would also revise, and so no agent revises. If s is small, then the implied delay is not too costly relative to the benefit of changing the project payoffs, even if the other agent also revises. Then, continuing the project is preferable. The threshold at which s is too large to continue is higher for the agent with the higher probability of control in the future, as this agent expects a higher chance of

Figure 2: Equilibrium Revision Strategies given s and w



(a) $r > \frac{1}{2}$



(b) $r < \frac{1}{2}$

Note: Dashed lines depict σ^A and solid lines depict σ^B given $r = 0.6$ (Panel a) and $r = 0.4$ (Panel b), and $v = 3, q = 0.25, p = 0.5, w = 1$.

reaching execution of her preferred project type. Hence, her expected payoff relative to the running cost is higher.

The MPE in each period $t \geq 1$ is unique outside the $(\bar{s}_2, \bar{s}_1)$ interval. The region of equilibrium multiplicity is the region where profitability of one's revisions depends on the other agent's revision strategy. If only one agent revises, while the other does not, the expected delays are not too costly relative to their expected benefit; however, the expected cost would be too high if both agents were to revise or the expected cost would be too low if both agents were to continue. As we will show below, our qualitative results do not depend on the equilibrium selection in this multiplicity region.

5.2 How Bureaucratic Capacity Feeds Bank into Project Setup

Given the predicted revision response to the project, agent A in period 0 chooses the scale s and division w (i.e., the inequality Δ) in order to maximize her expected utility given that the starting project is of type Δ^A:

$$EU^A(s,w) = r \cdot U^A(A, \Delta^A) + (1-r) \cdot U^A(B, \Delta^A). \tag{10}$$

Given the strategies described in Lemma 1 the expected utility in any of the pure strategy equilibria can be be expressed as

$$EU^A(s,w) = [H_1(q,p,r) \cdot w + H_2(q,p,r) \cdot (1-w)] \cdot s \cdot v - \frac{c(s)}{p} \cdot H_3(q,p,r), \tag{11}$$

where $H_1(q,p,r), H_2(q,p,r)$ and $H_3(q,p,r)$ are functions of q,p,r, with expressions given in the Appendix. They parameterize, respectively, the probability of agent A obtaining fraction w of the project benefits, the probability of this agent obtaining fraction $1-w$, and the expected delay in the project's completion. This formulation shows that the expected utility is piecewise linear in w and concave in s. Our main results describe how project characteristics are determined by the cost structure and bureaucratic capacity. We first show that scale is strategically used by the project initiator to preclude revisions:

Proposition 1 (Equilibrium Revisions) *There exists upper bound $\overline{\varepsilon}(s) \geq 4$ on the elasticity of the cost function $c(s)$ such that for $\varepsilon(s) \leq \overline{\varepsilon}(s)$, there are no revisions on the equilibrium path and the equilibrium project type is Δ^A.*

The project's initiator optimally designs its characteristics to avoid revisions down the line. The two tools at her disposal are the project scale and its division of benefits. In order to use these tools to discourage revisions, the cost of increasing scale must not increase too fast. Otherwise, any attempt to strategically increase scale beyond the point at which revisions are desirable would also make the entire project too expensive to build, even for politician A. Before discussing the construction of the equilibrium, we also state how project characteristics are mediated by bureaucratic organizational capacity.

Proposition 2 (Scale and Payoff Inequality) *When $\varepsilon(s) \leq \overline{\varepsilon}(s)$, the equilibrium scale s^* and payoff division w^* depend on p relative to two thresholds of q:*

- *(Unconstrained regime) If $p > \overline{q}(\varepsilon, r, q)$, then $w^* = 1$ and $s^* = s^{NT}$.*
- *(Over-scaling regime) If $p \in [q, \overline{q}(\varepsilon, r, q)]$, then $w^* \leq 1$ and $s^* > s^{NT}$.*
- *(Under-scaling regime) If $p < q$, then $w^* < 1$ and $s^* < s^{NT}$.*

The threshold $\overline{q}$ takes the following form:[7]

$$\overline{q} = \begin{cases} q \cdot \left(\varepsilon(\overline{s}_3) - \frac{2r}{1-r}\right) \text{ if } r \geq \frac{1}{2} \\ q \cdot \varepsilon(\overline{s}_1) \text{ if } r < \frac{1}{2} \end{cases} . \tag{12}$$

Notice that these expressions take a very simple form under a quadratic cost function: if $r \geq 1/2$, then $\overline{q} < 0$, whereas if $r < 1/2$, then $\overline{q} = 2q$.

The project's initiator optimally designs its characteristics to avoid revisions down the line. The two tools at her disposal are the project scale and its division of benefits. How these tools should be wielded depends on what is likely to happen once the project is under way. This in turn is a function of the bureaucracy's capacity to move the project along, captured by p, and of the likelihood that the project is successfully revised by a challenger. To show the trade-offs involved in using scale or payoff inequality strategically, it is helpful to consider them sequentially.

The expected duration of the project, and hence its expected running cost is determined directly by the bureaucracy's organizational capacity. When capacity is high, the expected run time is short, and therefore the revision-deterring benefit of a large scale outweighs the increase in running costs. This allows agent A to choose a large scale, and this alone is

[7]These expressions are for the case when the equilibrium selected in the region of multiplicity is not $(\sigma^A, \sigma^B) = (1, 0)$ if $r < 1/2$. The expressions if the alternative equilibrium is selected in the multiplicity region are slightly more complicated are given in the Appendix.

enough to deter revisions, without the need to compromise on w. In fact, the project scale can be as large as the one chosen by the agent in the benchmark without transitions of control.

As capacity decreases, the expected project run time and associated costs increase. How much revisions pose a threat, and therefore how much scale must be increased to deter them depends on how likely the opposition is to trigger them. If the probability that the opposition takes control is small ($r > 1/2$), then the revision threat remains sufficiently low such that scale alone deters the danger, even when set at agent $A's$ ideal.

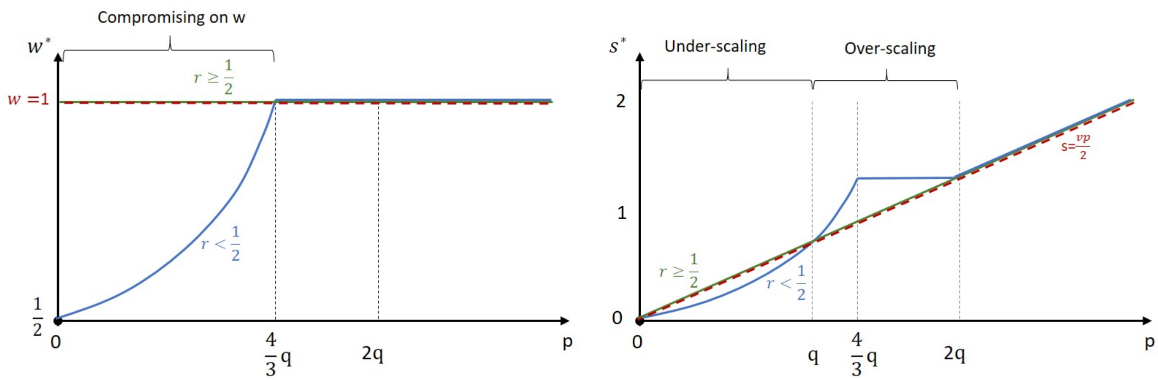
The calculus changes when the probability that the opposition takes control is large ($r < 1/2$). With intermediate capacity, the expected run costs are higher, and agent A would ideally reduce the scale to adjust for these higher costs. Yet, in order to deter revisions by B, she must keep the scale large enough. This results in over-scaling of the project, so that $s^* > s^{NT}$. Finally, if capacity drops even more, setting a large project scale in order to deter revisions becomes too costly, as the project is expected to run for a long time in stage d. Agent A can save on scale increases by compromising on w as well. She offers B enough benefits so as to reduce the gains from a revision. As she gives away more benefits to B, the relative cost of running a large scale project further increases. This drives to under-scaling of the project relative to agent $A's$ unconstrained ideal s^{NT}. We illustrate the equilibrium w and s in Figure 3.

Proposition 2 implies that higher capacity increases project scale but also the inequality in payoff division.

Corollary 1 (Effect of Higher Capacity) *Higher organizational capacity p increases equilibrium scale s and payoff inequality Δ.*

As project run times are expected to be faster, the project initiator harness capacity to her advantage: she uses it to make projects larger and to divide the benefits more unequally.

Figure 3: Equilibrium Project Characteristics



Note: Equilibrium w (left panel) and s (right panel) for $r = 0.6$ (green), $r = 0.4$ (blue), and $v = 5, q = 0.25$. The red dashed line shows the scale and the payoff division under no transitions.

5.3 Project Revisions in Equilibrium

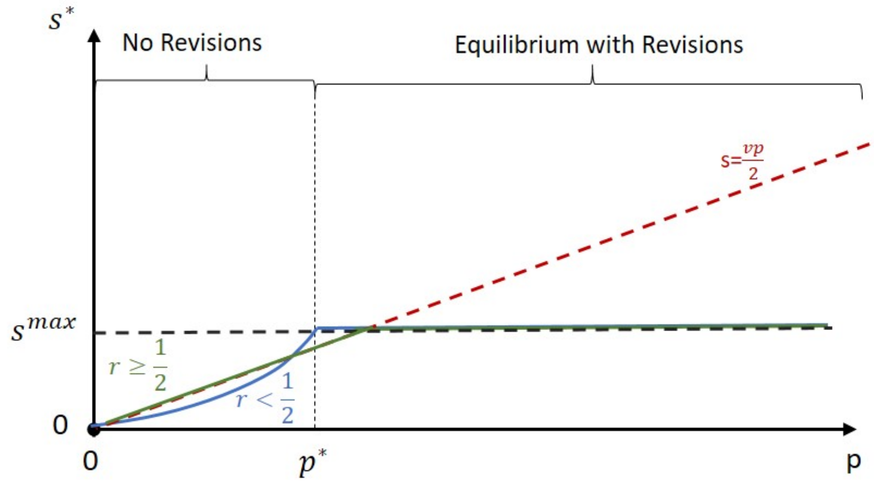
So far, we have shown what happens when agent A has full flexibility to scale the project, and she uses this tool in order to deter revisions in equilibrium. She can successfully deploy this tool as long as the cost structure allows relatively affordable scaling, as shown in Proposition 1. Yet, if the maximum scale of the project is limited, either by budget rules or by a highly convex cost structure, then revisions may not be avoidable on the equilibrium path.

First, consider the case where the maximum project for the project, $s^{\max}$, is not large enough to accommodate the needed over-scaling. Then, revisions cannot be avoided.

Proposition 3 (Limited Scale and Equilibrium Revisions) *There exists threshold value $s^M(p)$ such that, if $s^{\max} \leq s^M(p)$, then the equilibrium project scale is $s^{\max}$ if capacity is at least p, the equilibrium payoff inequality is maximal, $\Delta = 1$, and there are revisions on the equilibrium path: each incumbent revises a project favorable to their opponent.*

The result is illustrated in Figures 4. Agent A uses scale to strategically deter revisions, up until the needed scale reaches the ceiling $s^{\max}$. At that point, the strategic use of scale is exhausted. Moreover, if the ceiling $s^{\max}$ is sufficiently low, the cost of delays is negligible

Figure 4: Limited Scale and Revision



Note: Equilibrium s for $r = 0.6$ (green), $r = 0.4$ (blue), and $v = 5, q = 0.25$. The red dashed line shows the scale and the payoff division under no transitions. The back dashed line shows the scale upper limit $s^{\max}$.

relative to the potential gain from a revision. In this case, any compromise on the payoff division in order to deter revisions would have to be exceedingly large. Agent A then prefers to claim all benefits for herself and enter a 'winner-take-all' regime where everyone revises the project in their favor.

Corollary 2 (Higher Capacity under Scale Constraints) *If project scale is limited, higher organizational capacity* p *increases the probability of project revisions and delays.*

While p is small, agent A is under-scaling and compromising, such that both s and w are reduced in order to avoid revisions. Yet, as p increases, revisions become less costly. To discourage them, agent A would have to over-scale the project. As she hits the scale limit $s^{\max}$, revisions are no longer avoidable. The game switches to a contest with maximal inequality in payoff division, where each incumbent attempts to revise their opponent's project. This equilibrium reduces agent $A's$ welfare relative to the outcome without scale limitations.

Another potential driver of revisions in equilibrium is a cost structure that makes over-scaling unsustainable. If $\varepsilon(s)$ is very large, then increasing s in order to deter revisions is too costly, and revisions cannot be avoided in equilibrium.

Proposition 4 (Cost Structure and Equilibrium Revisions) *For large $\varepsilon(s)$, then the equilibrium payoff inequality is maximal, $\Delta = 1$, and there are revisions on the equilibrium path: each incumbent revises a project favorable to their opponent.*

A cost function with high elasticity limits how much politician A can increase scale. A small increase in scale is sufficient to inflate the running costs beyond what is desirable for both agents. Politician A prefers the alternative of starting a small project, but trying to capture its entire benefit by setting $\Delta = 1$. Each transition of control then triggers a revision of an unfavorable project, leading to long completion timelines.

5.4 Welfare

Our results so far show that organizational capacity has pronounced effects on the strategies of project designers. Higher values of capacity increase inequality, while low and medium values result in under- and over-scaling. These strategies suggest significant implications for social benefits. In particular, when capacity lies in the interval $[q, \overline{q}]$, projects are both over-scaled and highly unequal, and are therefore especially harmful to the non-initiating agent.

We investigate aggregate benefits for the special case of quadratic costs ($c(s) = s^2$). This calculation first requires an expression for ex ante expected payoffs. Using the results from

Proposition 2, agent A's expected payoff prior to the revelation of the initiator evaluates to:

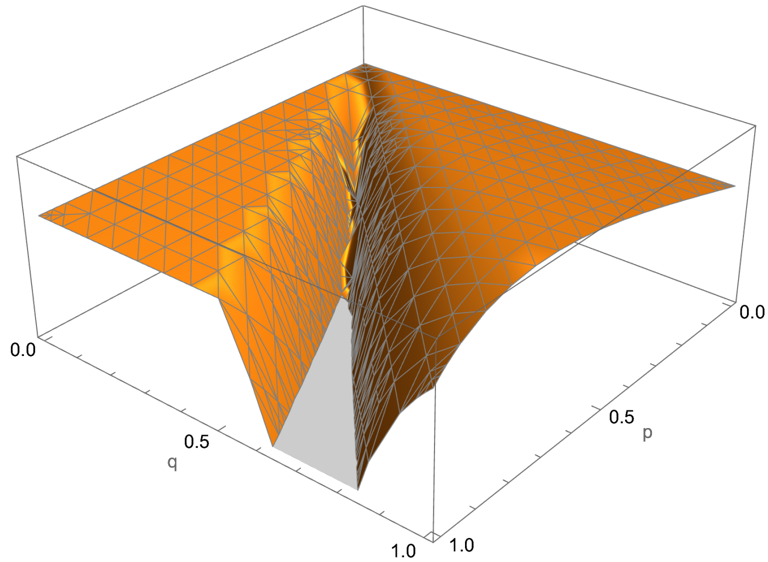
$$EU^A(s^*,\Delta^*) = \begin{cases} \frac{pv^2\left[2p^2r - pq(5r+3) + 2q^2(3r+1)\right]}{8(p-2q)^2} & \text{if } p < \frac{4}{3}q \text{ and } r > 1/2 \\ \frac{pv^2[q(4-5r)-2p(1-r)]}{8(p-2q)} & \text{if } p < \frac{4}{3}q \text{ and } r < 1/2 \\ \frac{v^2\left[p^2r - 4q^2(1-r)\right]}{4p} & \text{if } p \in \left[\frac{4}{3}q, \overline{q}\right) \text{ and } r > 1/2 \\ \frac{v^2\left[4qr(p-q) - p^2(1-r)\right]}{4p} & \text{if } p \in \left[\frac{4}{3}q, \overline{q}\right) \text{ and } r < 1/2 \\ \frac{pv^2(2r-1)}{4} & \text{if } p \geq \overline{q}. \end{cases} \tag{13}$$

Our measure of welfare simply sums the expressions in (13) for each range of p and each agent, where agent B's expected payoff equals agent A's payoff under the complementary probability of holding power. The sum is weighted by the probability of being the initiator. Without loss of generality, for $r > 1/2$ this produces:

$$\begin{cases} \frac{pv^2\left[2p^2r(2r-1) - pq(14r^2-7r+1) + 2q^2(8r^2-5r+1)\right]}{8(p-2q)^2} & \text{if } p < \frac{4}{3}q \\ \frac{v^2\left[p^2r(2r-1) + 4pq(1-r)^2 - 4q^2(1-r)\right]}{4p} & \text{if } p \in \left[\frac{4}{3}q, \overline{q}\right) \\ \frac{pv^2(2r-1)^2}{4} & \text{if } p \geq \overline{q}. \end{cases} \tag{14}$$

Figure 5 illustrates welfare as a function of p and q. It shows that the over-scaling regime is especially bad for aggregate payoffs. As the values of q corresponding to this regime increases with capacity, the implication is that systems with high institutional barriers and high capacity are prone to producing poor projects. By contrast, systems with low capacity and high barriers, or high capacity and low barriers, produce projects that are socially beneficial, despite possible distributional or scale deficiencies.

Figure 5: Welfare



Note: Welfare as a function of p and q. Parameters are $r = 0.52$ and $v = 1$.

6 Multiple Phases

We now adapt the preceding results to a model with two phases. In the basic model, period 0 is distinguished by the ability of the project initiator to choose key program parameters. Inherently complex projects such as those often funded by FTA Capital Improvement Grants typically present multiple opportunities for politicians to revisit basic questions of scale and distribution. For example, in 2011 the Obama administration proposed the $30 billion Gateway Program to upgrade rail infrastructure between New York and New Jersey. Despite favorable FTA reviews, the Trump administration effectively cancelled the program, only to have it revived under the Biden administration.[8]

Complex projects often require advance research and planning, and therefore early phases of such projects correspond naturally to investments that reduce subsequent construction or implementation costs. These investments may also provide benefits in their own right,

[8]See Matt Hickman, "New York and New Jersey's long-delayed Gateway Program faces a more favorable outlook under Biden presidency." *The Architect's Newspaper*, November 10, 2020.

independently of the final project outcome. It is therefore worth asking how the possibility of resetting program parameters mid-stream affects investments, project scale, and revisions. In particular, we examine conditions under which transitions of power may prevent projects from starting at all.

Each phase of the two-phase model is structurally identical to the basic model. Agent A holds office at the start of phase 1, and holds office at the start of phase 2 with probability r. Denote the parameters for scale, distribution, and valuation in phase τ by s_τ, Δ_τ, and v_τ, respectively. As in the basic model, s_τ and Δ_τ are chosen in the initial incumbent in each phase, v_τ is exogenous, and project types are determined by players after the initial period of the phase. The phase 1 payoffs thus represent the immediate value of investments such as research contracts or pilot studies. To keep the analysis tractable, when there are multiple equilibria we select the one in which only the favored agent revises.

The phases are dynamically linked through their cost functions. Let the cost of each period in phase τ be $c(s_\tau) = m_\tau s_\tau^2$, where $m_\tau > 0$ and $m_1 = 1$. In phase 2, $m_2 = 1/s_1$, so that early investments in the project reduce future marginal costs. Note that in isolation, phase 1 of the model is identical to the basic game if $s_2 = 0$, and phase 2 of the model is identical to the basic game if $s_1 = 1$.

Within each phase τ, actions following the choice of s_τ only affect payoffs through the division of v_τ. Thus, the agents' incentives following the initial period are similar to those of the one-phase game, and we can exploit the derivations of Section 5 to analyze revisions and the choice of Δ_τ. The second phase primarily affects agent A's incentives in choosing the phase 1 scale, which affects phase 2 costs. Due to the simple structure of m_2 and quadratic costs, s_1 linearly scales A's phase 2 expected payoff. Her phase 1 objective can be expressed as:

$$EU^A(s_1, \Delta_1) + s_1 \tilde{U}_2^A, \tag{15}$$

where $\tilde{U}_2^A = EU^A(s^*, \Delta^*)$ is agent A's phase 2 expected payoff prior to the revelation of the phase 2 initiator. The expression for $\tilde{U}_2^A$ is identical to that for $\tilde{U}^A$ (expression (13)), with the exception of substituting v_2 for v.

Maximizing (15) with respect to s_1 produces our next result. Roughly speaking, the phase 1 investment is the scale of the one-phase game, s^*, adjusted to reflect $\tilde{U}^A$. Importantly, $\tilde{U}^A$ is negative whenever $r < 1/2$, as well as for some values of p between q and $\overline{q}$ (where $\overline{q} = 2q$ under quadratic costs) when $r > 1/2$. When this happens, the phase 1 scale s_1^* is lower than s^*. Consistent with Lemma 1, s_1^* may even be low enough to induce revisions in equilibrium. Negative values of $\tilde{U}^A$ play a role similar to that of increasing the cost of high project scales in the one-phase model: inhibiting large scales generates projects that are insufficient to deter revisions.

Beyond merely reducing scale, the optimal scale in the initial phase may be zero, which in effect cancels the project. Proposition 5 provides conditions under which this occurs.

Proposition 5 (Two Phases) *If $r > 1/2$, then $s_1^* = 0$ only if $p \in [q, \overline{q}]$ and if:*

$$\tilde{U}^A < -\frac{rv\left[p(1-r)+qr\right]}{p(1-r)r+q\left(2r^2-2r+1\right)}. \tag{16}$$

If $r < 1/2$, $s_1^* = 0$ if v_1 is sufficiently low or v_2 is sufficiently high.

For a favored ($r > 1/2$) phase 1 initiator, cancellations occur because of the potential for over-scaling. As Figure 3 illustrates, under moderate capacity an unfavored agent B over-scales to prevent revisions. This can produce a highly undesirable expected payoff for agent A, especially if she is not overwhelmingly likely to retain power. A highly competitive political environment thereby encourages cancellations by forcing agent A to internalize the social benefits of the project.[9] By contrast, under low capacity, under-scaled projects are

[9]Note however that public projects may provide public good benefits to actors besides agents A and B.

relatively efficient and do not invite cancellation. And under high capacity, a favored initiator is likely to benefit from an unequal phase 2 project.

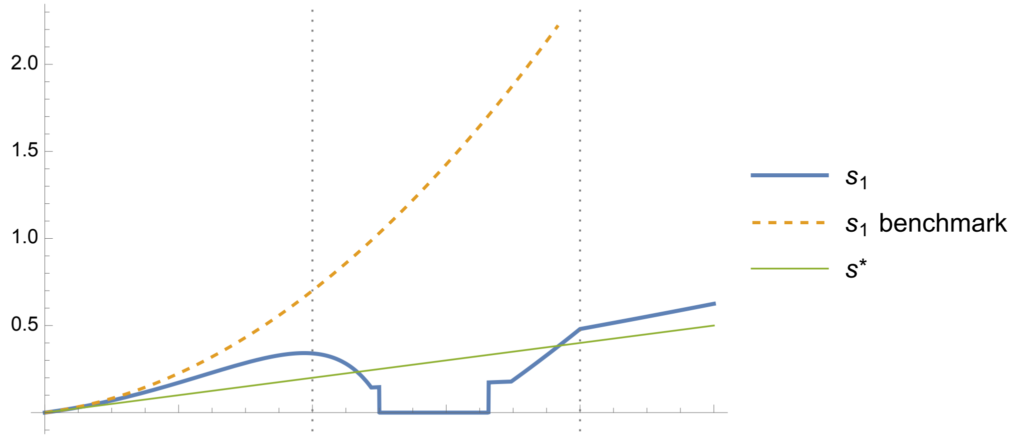
For an unfavored ($r < 1/2$) phase 1 initiator, the main driver of cancellation is the distribution of payoffs over time. Phase 1 produces positive expected payoffs for the initiator, but phase 2 produces negative ex ante expected payoffs at any capacity level. Thus she will simply cancel if v_2 is high relative to v_1.

Figure 6 illustrates the role of cancellations by comparing phase 1 investments against two benchmarks in the $r > 1/2$ case. In the first benchmark, A remains in power with certainty at the beginning of phase 2, but faces the possibility of revision in both phases. As expected, the possibility of losing control over the final project depresses investment. The second benchmark is simply the equilibrium scale s^{NT} in the one-phase game. The initial investment s_1^* may be under- or over-scaled relative to this benchmark, depending on agent A's expected phase 2 payoffs. In this example, power transitions are very likely ($p = 0.52$), so the threat of over-scaling by B in phase 2 causes under-scaling and cancellations when capacity is in the interval $[\underline{q}, \overline{q}]$. This non-monotonicity of project scale with respect to capacity reflects the non-monotonicity of social benefits in the one-phase game, as illustrated in Figure 5.

7 Conclusion

Within academic and policy circles, bureaucratic capacity has become a hallmark of good governance. But in contrast to the consensus about its benefits, there is little agreement on its practical definition, and also little work that explores its implications for key features of public policies. Our theory addresses both of these issues. It models capacity as the transition probability of a simple Markov process, and then situates this process in an in-

Figure 6: Investment with Two Phases



Note: Initial investment (s_1, blue), benchmark investment (dashed) in a setting with where A chooses s_2 and Δ_2 in phase 2, and investment in the one-phase game ($s^* = s^{NT}$, green), as a function of p. Parameters are $r = 0.52$, $v_1 = 1$, $v_2 = 5$, and $q = 0.4$. Vertical lines are located at the thresholds q and $\overline{q}$, between which over-scaling may occur.

stitutional environment that features political contestation and institutional rigidities. This basic framework allows us to capture a rich set of outputs, such as the scale, timing, and distributive properties of large public projects.

The principal equilibrium incentive in the model is the avoidance of revisions, which can delay completion and reduce payoffs. Depending on capacity levels, this produces political incentives to manipulate the scale and distributive properties of projects. Several unexpected and potentially testable implications follow from this incentive. First, by reducing the opportunities for obstruction, high capacity agencies encourage larger and less egalitarian projects. Second, when project scales are constrained, the inability to over-scale projects causes greater revision and delay as capacity increases. Third, "moderate" capacity levels that encourage over-scaling produce poor projects from a social welfare perspective. Finally, in complex multi-phase projects, potential transitions of power can result in project cancellations. In short, greater capacity does not unambiguously improve government performance.

Our model treats organizational capacity as exogenous, but its implications for policy performance raise some basic questions about its origins. We mention two as possibilities for further inquiry. First, just as recent work on state capacity has explored the political and economic drivers of investment in taxing powers, it is worth examining politicians' incentives to invest in the capabilities of agencies that may far outlive them. Second, it may be useful to unpack the capacity parameter p to reflect the needs of modern projects. For example, outside contractors often play major roles in major infrastructure construction, but whether such players enhance capacity, or are symptoms of low capacity, is unclear.[10]

[10] See Ralph Vartabedian, "How California's faltering high-speed rail project was 'captured' by costly consultants." *Los Angeles Times*, April 26, 2019.

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Appendix

A Proofs

A.1 Proof of Lemma 1

Given the Markov transition probabilities, the value function for Politician A starting in the state where control is held by Politician $i \in \{A, B\}$ and the project type is Δ^j with $j \in \{A, B\}$ is

$$U^A(i, \Delta^j | \sigma^A, \sigma^B) = sv \cdot [H_1^{ij}(p,q,r) \cdot w + H_2^{ij}(p,q,r) \cdot (1-w)] - \frac{c(s)}{p} \cdot H_3^{ij}(p,q,r) \cdot, \quad (17)$$

while the corresponding utility for Politician B is

$$U^B(i, \Delta^j | \sigma^A, \sigma^B) = sv \cdot [H_1^{ij}(p,q,r) \cdot (1-w) + H_2^{ij}(p,q,r) \cdot w] - \frac{c(s)}{p} \cdot H_3^{ij}(p,q,r) \cdot, \quad (18)$$

where $H_1^{ij}(p,q,r) = \frac{\Gamma_{ij}(p,q,r)}{\Omega}$, $H_2^{ij}(p,q,r) = \frac{\Upsilon_{ij}(p,q,r)}{\Omega}$, and $H_3^{ij}(p,q,r) = \frac{\Sigma_{ij}(p,q,r)}{\Omega}$, and

$$\Omega = p(1 - r\sigma^A)(1 - (1-r)\sigma^B) + q(r\sigma^A + (1-r)\sigma^B) - 2qr(1-r)\sigma^A\sigma^B \quad (19)$$

$$\Sigma_{AA} = p(1 - p(1-r)\sigma^B)(1 - r\sigma^A) - 2qp\sigma^A\sigma^B r(1-r) + q(r\sigma^A + (1-r)\sigma^B) \quad (20)$$

$$\Sigma_{AB} = p(1 + p(1-r)\sigma^A)(1 - (1-r)\sigma^B) + 2qp\sigma^A\sigma^B(1-r)^2 \quad (21)$$

$$+ q(r\sigma^A + (1-r)\sigma^B) \quad (22)$$

$$\Sigma_{BA} = p(1 + pr\sigma^B)(1 - r\sigma^A) + 2qp\sigma^A\sigma^B r^2 + q(r\sigma^A + (1-r)\sigma^B) \quad (23)$$

$$\Sigma_{BB} = p(1 - pr\sigma^A)(1 - (1-r)\sigma^B) - 2qp\sigma^A\sigma^B r(1-r) + q(r\sigma^A + (1-r)\sigma^B) \quad (24)$$

$$\Gamma_{AA}(p,q,r) = p(1 - r\sigma^A)(1 - (1-q)(1-r)\sigma^B) + qr\sigma^A(1 - (1-r)\sigma^B) \quad (25)$$

$$\Gamma_{AB}(p,q,r) = p(1 - (1-r)\sigma^B)q(1-r)\sigma^A + qr\sigma^A(1 - (1-r)\sigma^B) \quad (26)$$

$$\Gamma_{BA}(p,q,r) = p(1 - r\sigma^A)(1 - qr\sigma^B - (1-r)\sigma^B) + qr\sigma^A(1 - (1-r)\sigma^B) \quad (27)$$

$$\Gamma_{BB}(p,q,r) = p(1 - (1-r)\sigma^B)qr\sigma^A + qr\sigma^A(1 - (1-r)\sigma^B) \quad (28)$$

$$\Upsilon_{AA}(p,q,r) = -pq(1-r)\sigma^B(1 - r\sigma^A) + q(1-r)\sigma^B(1 - r\sigma^A) \quad (29)$$

$$\Upsilon_{AB}(p,q,r) = p(1 - r\sigma^A - q(1-r)\sigma^A)(1 - (1-r)\sigma^B) + q(1-r)\sigma^B(1 - r\sigma^A) \quad (30)$$

$$\Upsilon_{BA}(p,q,r) = -pqr\sigma^B(1 - r\sigma^A) + q(1-r)\sigma^B(1 - r\sigma^A) \quad (31)$$

$$\Upsilon_{BB}(p,q,r) = p(1 - (1-r)\sigma^B)(1 - (1-q)r\sigma^A) + q(1-r)\sigma^B(1 - r\sigma^A) \quad (32)$$

Given $\sigma^A, \sigma^B \in \{0, 1\}$, we can have a pure strategy equilibrium if each agent $i \in \{A, B\}$ prefers to follow her prescribed strategy given the other agent j's strategy. For agent i, if

$\sigma^i = 1$, then the payoff from revision is

$$U^{i,R} = rqU^i(i,\Delta^i) + r(1-q)U^i(i,\Delta^j) \\ + q(1-r)U^i(j,\Delta^i) + (1-r)(1-q)U^i(j,\Delta^j).$$

If $\sigma^i = 0$, then the payoff from project continuation of a project Δ^j is

$$U^{i,C} = psv(1-w) + (1-p)(1-r)U^i(j,\Delta^j) + (1-p)rU^i(i,\Delta^j)$$

Case 1: $\sigma^A = 1$ and $\sigma^B = 1$. This is an equilibrium if $U^{i,R} \geq U^{i,C}$ for $i \in \{A,B\}$. These conditions reduce to two upper bounds on s, such that this equilibrium is sustainable if

$$\frac{c(s)}{s} \in \left[0, qv(2w-1)\cdot \min\left\{\frac{p(1-r)}{p(1-r)+2qr}; \frac{pr}{pr+2q(1-r)}\right\}\right].$$

Case 2: $\sigma^A = 1$ and $\sigma^B = 0$. This is an equilibrium if

$$U^{A,R}(A,\Delta^B) \geq U^{A,C}(A,\Delta^B),$$
$$U^{B,R}(B,\Delta^A) < U^{B,C}(B,\Delta^A).$$

Each of the above conditions reduces to a threshold on s:

$$\frac{c(s)}{s} \leq qv(2w-1),$$
$$\frac{c(s)}{s} \geq qv(2w-1)\frac{p(1-r)}{p(1-r)+2qr}.$$

Therefore, the equilibrium exists for

$$\frac{c(s)}{s} \in \left[qv(2w-1)\frac{p(1-r)}{p(1-r)+2qr}, qv(2w-1)\right].$$

Case 3: $\sigma^A = 0$ and $\sigma^B = 1$. This is an equilibrium if

$$U^{A,R}(A,\Delta^B) < U^{A,C}(A,\Delta^B),$$
$$U^{B,R}(B,\Delta^A) \geq U^{B,C}(B,\Delta^A).$$

Each of the above conditions reduces to a threshold on s:

$$\frac{c(s)}{s} \geq qv(2w-1)\frac{pr}{pr+2q(1-r)},$$
$$\frac{c(s)}{s} \leq qv(2w-1).$$

Therefore, the equilibrium exists for

$$\frac{c(s)}{s} \in \left[qv(2w-1)\frac{pr}{pr+2q(1-r), qv(2w-1)}\right].$$

Case 4: $\sigma^A = 0$ and $\sigma^B = 0$. This is an equilibrium if

$$U^{A,R}(A, \Delta^B) < U^{A,C}(A, \Delta^B),$$
$$U^{B,R}(B, \Delta^A) < U^{B,C}(B, \Delta^A).$$

Each of the above conditions reduces to a threshold on $\frac{c(s)}{s}$ of $\frac{c(s)}{s} \geq qv(2w-1)$ for both politicians. Therefore, the equilibrium exists for $\frac{c(s)}{s} \geq qv(2w-1)$.

Case 5. Mixed Strategy Equilibria. For mixed strategy equilibria, consider the case where Politician A mixes with $\sigma^A \in (0,1)$, while $B's$ strategy is $\sigma^B \in [0,1]$. This requires $U^A(A, \Delta^B|1, \sigma^B) = U^A(A, \Delta^B|0, \sigma^B)$, and thus

$$\sigma^B = \frac{p(s - qv(2w-1))}{(1-r)\left[p(s - qv(2w-1)) - 2q\frac{c(s)}{s}\right]}. \tag{33}$$

Similarly, Politician B mixes while $A's$ strategy is σ^A if $U^B(B, \Delta^B|\sigma^A, 1) = U^B(B, \Delta^B|\sigma^B, 0)$, and thus

$$\sigma^A = \frac{p(\frac{c(s)}{s} - qv(2w-1))}{r\left[p(\frac{c(s)}{s} - qv(2w-1)) - 2q\frac{c(s)}{s}\right]}. \tag{34}$$

The condition for mixing is that $\sigma^B \in [0,1]$ and $\sigma^A \in [0,1]$. Given (33) and (34), this implies

$$\frac{c(s)}{s} \in \left[pqv(2w-1)\max\left\{\frac{1-r}{p(1-r)+2qr}, \frac{r}{pr+2q(1-r)}\right\}, qv(2w-1)\right]. \tag{35}$$

Notice that the above condition allows for an equilibrium with $\sigma^A = 1, \sigma^B \in (0,1)$ if $\frac{c(s)}{s} = \frac{1-r}{p(1-r)+2qr}$ and $\max\left\{\frac{1-r}{p(1-r)+2qr}, \frac{r}{pr+2q(1-r)}\right\} = \frac{1-r}{p(1-r)+2qr}$. Conversely, an equilibrium with $\sigma^B = 1, \sigma^A \in (0,1)$ exists if $\frac{c(s)}{s} = \frac{r}{pr+2q(1-r)}$ and $\max\left\{\frac{1-r}{p(1-r)+2qr}, \frac{r}{pr+2q(1-r)}\right\} = \frac{r}{pr+2q(1-r)}$.

A.2 Proof of Propositions 1 and 2

We begin by establishing the following auxiliary results.

Auxiliary Lemma 1 *Politician A's expected utility at time 0 is piecewise linear in w for $w \in [0,1]$.*

Proof. Given (17) and (18), with pure strategy equilibria, $\sigma^A, \sigma^B \in \{0,1\}$,

$$\frac{\partial^2 U^A(i,\Delta^j|\sigma^A,\sigma^B)}{\partial w^2} = \frac{\partial^2 U^B(i,\Delta^j|\sigma^A,\sigma^B)}{\partial w^2} = 0.$$

For mixed strategy equilibria, we have

$$\begin{aligned}\frac{\partial U^A(A,\Delta^A|\sigma^A,\sigma^B)}{\partial w} &= -(1-2p)sv,\\ \frac{\partial U^A(B,\Delta^A|\sigma^A,\sigma^B)}{\partial w} &= -\frac{1-(1-2p)r}{1-r}sv.\end{aligned}$$

Thus, also with mixed strategy equilibria,

$$\frac{\partial^2 U^A(i,\Delta^j|\sigma^A,\sigma^B)}{\partial w^2} = \frac{\partial^2 U^B(i,\Delta^j|\sigma^A,\sigma^B)}{\partial w^2} = 0.$$

Since $EU^A(s,w) = r \cdot U^A(A,\Delta^A) + (1-r) \cdot U^A(B,\Delta^A)$, it follows that $EU^A(s,w)$ is linear in w given σ^A, σ^B. ■

Auxiliary Lemma 2 *Politician A's expected utility is increasing in w whenever $\sigma^B = 0$.*

Proof. In period 0, the expected utility is

$$\begin{aligned}EU^A(s,w|\sigma^A,\sigma^B) = sv \cdot \Bigg[&\left(r\frac{\Gamma_{AA}(p,q,r)}{\Omega} + (1-r)\frac{\Gamma_{BA}(p,q,r)}{\Omega}\right)w\\ &+ \left(r\frac{\Upsilon_{AA}(p,q,r)}{\Omega} + (1-r)\frac{\Upsilon_{BA}(p,q,r)}{\Omega}\right)(1-w)\Bigg]\\ &- \frac{c(s)}{p} \cdot \left(\frac{\Sigma_{AA}(p,q,r)}{\Omega} + \frac{\Sigma_{BA}(p,q,r)}{\Omega}\right),\end{aligned}$$

which expands to

$$\begin{aligned}EU^A(s,w|\sigma^A,\sigma^B) = &\; s \cdot \frac{vp}{\Omega} \cdot (1-r\sigma^A) \cdot (1-(1-r)\sigma^B) \cdot w\\ &+ s \cdot \frac{vq}{\Omega} \cdot [r\sigma^A w + (1-r)\sigma^B(1-w) - r(1-r)\sigma^A\sigma^B]\\ &- \frac{c(s)}{p\Omega}\left[q(r\sigma^A + (1-r)\sigma^B) + p(1-r\sigma^A)\right]. \quad (36)\end{aligned}$$

It follows that if $\sigma^A = \sigma^B = 0$, or if $\sigma^A = 1, \sigma^B = 0$ then

$$EU^A(s,w) = svw - \frac{c(s)}{p}, \quad (37)$$

and thus

$$\frac{\partial EU^A(s,w)}{\partial w} = sv > 0.$$

■

Auxiliary Lemma 3 *Politician A's expected utility is monotone, either increasing or decreasing in w, whenever $\sigma^B = 1$.*

Proof. If $\sigma^A = 0, \sigma^B = 1$, then

$$EU^A(s,w|(0,1)) = sv\frac{q(1-r)(1-w)+prw}{q(1-r)+pr} - \frac{c(s)}{p}\frac{q(1-r)+p}{q(1-r)+pr},$$

and thus

$$\frac{\partial EU^A(s,w)}{\partial w} = sv\frac{pr-q(1-r)}{pr+q(1-r)}, \tag{38}$$

which means

$$\frac{\partial EU^A(s,w)}{\partial w}\begin{cases} >0 & \text{if } p > q\frac{1-r}{r} \\ <0 & \text{if } p < q\frac{1-r}{r} \end{cases}.$$

If $\sigma^A = 1, \sigma^B = 1$, then

$$EU^A(s,w|(1,1)) = sv\frac{q(1-r)^2+(q(2r-1)+p(1-r)r)w}{q(1-2r(1-r))+pr(1-r)} - \frac{c(s)}{p}\frac{p(1-r)+q}{q(1-2r(1-r))+pr(1-r)},$$

and thus

$$\frac{\partial EU^A(s,w)}{\partial w} = sv\frac{q(2r-1)+pr(1-r)}{q(1-2r(1-r))+pr(1-r)}, \tag{39}$$

which means

$$\frac{\partial EU^A(s,w)}{\partial w}\begin{cases} >0 & \text{if } p > q\frac{2r-1}{r(1-r)} \\ <0 & \text{if } p < q\frac{2r-1}{r(1-r)} \end{cases}.$$

■

Auxiliary Lemma 4 *Politician A's expected utility is monotone decreasing in w if the equilibrium is mixing.*

Proof. Given the equilibrium mixing probabilities σ^A, σ^B, we have

$$EU^A(s,w|(\sigma^A,\sigma^B)) = sv(1-w) - c(s)\left(\frac{1}{p}-\frac{1}{q}\right).$$

Hence,

$$\frac{\partial EU^A(s,w|(\sigma^A,\sigma^B))}{\partial w} = -sv < 0.$$

■

The values of $w(s)$ which separate the different equilibrium regions are $w \in \{0, w_1, w_2, w_3, 1\}$, where

$$w_1(s) = \frac{1}{2} + \frac{c(s)}{s}\frac{1}{2qv},$$
$$w_2(s) = \frac{1}{2} + \frac{c(s)}{s}\frac{1}{2qv} + \frac{c(s)}{s}\frac{1}{pv}\min\left\{\frac{1-r}{r}, \frac{r}{1-r}\right\},$$
$$w_3(s) = \frac{1}{2} + \frac{c(s)}{s}\frac{1}{2qv} + \frac{c(s)}{s}\frac{1}{pv}\max\left\{\frac{1-r}{r}, \frac{r}{1-r}\right\}.$$

Auxiliary Lemma 5 *Given any s, politician A's expected utility is continuous in w with the exception of a finite number of discontinuities:*

- *A jump down at w_3 if $r > 0.5$;*
- *A jump up at w_2 if the equilibrium selected when $w \in (w_1, w_2)$ is mixing or (i) $r > 0.5$ and $\sigma^A = 0, \sigma^B = 1$ or (ii) $r < 0.5$ and $\sigma^A = 1, \sigma^B = 0$.*
- *A jump down at w_1 if the equilibrium selected is $\sigma^A = 0, \sigma^B = 1$ when $w \in (w_1, w_2)$;*

Proof.

Case 1: $r > \frac{1}{2}$**.** In this case, $w_2 = \frac{1}{2} + \frac{c(s)}{2qvs} + \frac{c(s)}{pvs}\frac{1-r}{r}$, $w_3 = \frac{1}{2} + \frac{c(s)}{2qvs} + \frac{c(s)}{pvs}\frac{r}{1-r}$, and the equilibrium for $w \in (w_2, w_3)$ is $\sigma^A = 1, \sigma^B = 0$. Therefore, the equilibrium changes at w_3 from $\sigma^B = 0$ to $\sigma^B = 1$. We have

$$\lim_{w \uparrow w_3} EU^A(s, w|(0,0)) = \frac{sv}{2} + c(s)\left[\frac{1}{2q} + \frac{2r-1}{p(1-r)}\right],$$

and

$$\lim_{w \downarrow w_3} EU^A(s, w|(1,1)) = \frac{sv}{2} + \frac{c(s)}{2}\frac{1}{q}\frac{pr - q(1-r) + q\frac{3r^2}{1-r}}{pr + q(1-r) + q\frac{r^2}{1-r}} + \frac{c(s)}{p}\frac{q\frac{r^2-2r(1-r)}{(1-r)^2} - q - p}{pr + q(1-r) + q\frac{r^2}{1-r}}.$$

Then,

$$\lim_{w \uparrow w_3} EU^A(s, w) - \lim_{w \downarrow w_3} EU^A(s, w) = \frac{c(s)}{p}\frac{2(1-r)(p(1-r) + 2qr)}{q(1 - 2r(1-r)) + pr(1-r)} > 0.$$

If in the region of multiplicity the equilibrium selected is $\sigma^A = 1, \sigma^B = 0$, then the expected utility is continuous and has the same expression as a function of w for all $w \leq w_3$. If in the region of multiplicity the equilibrium selected is $\sigma^A = 0, \sigma^B = 1$, then the equilibrium

σ^B changes at both w_1 and w_2. At w_2 we have

$$\lim_{w\uparrow w_2} EU^A(s,w|(0,1)) = \frac{sv}{2} + \frac{c(s)}{2q} - \frac{c(s)}{pr},$$

$$\lim_{w\downarrow w_2} EU^A(s,w|(1,0)) = \frac{sv}{2} + \frac{c(s)}{2q} + \frac{c(s)}{p}\frac{1-2r}{r}.$$

Then,

$$\lim_{w\uparrow w_2} EU^A(s,w) - \lim_{w\downarrow w_2} EU^A(s,w) = -2\frac{c(s)}{p}\frac{1-r}{r} < 0. \tag{40}$$

Hence, there is a jump up at w_2.

At w_1, we have

$$\lim_{w\uparrow w_1} EU^A(s,w|(0,0)) = \frac{sv}{2} + \frac{c(s)}{2q} - \frac{c(s)}{p},$$

$$\lim_{w\downarrow w_1} EU^A(s,w|(0,1)) = \frac{sv}{2} - (1-r)\frac{c(s)}{2} + c(s)\frac{pr}{2q} - \frac{c(s)}{p}\frac{q(1-r)+p}{q(1-r)+pr}.$$

Then,

$$\lim_{w\uparrow w_1} EU^A(s,w) - \lim_{w\downarrow w_1} EU^A(s,w) = \frac{c(s)(1-pr+q(1-r))}{2q} + \frac{c(s)}{p}\frac{p(1-r)}{q(1-r)+pr} > 0. \tag{41}$$

Hence, there is a jump down at w_1.

If in the region of multiplicity the mixing equilibrium is selected, then at w_1, $\sigma^A = 1, \sigma^B = 0$, and therefore $EU^A(s,w_1|(0,0)) = EU^A(s,w_1|(\sigma^A,\sigma^B))$. Hence, there is no discontinuity at w_1. At w_2, $\sigma^A = 0, \sigma^B = 1$, and therefore $EU^A(s,w_1|(0,1)) = EU^A(s,w_1|(\sigma^A,\sigma^B))$. Hence, there is the same discontinuity at w_2 as in (40).

Case 2: $r < \frac{1}{2}$. In this case, $w_2 = \frac{1}{2} + \frac{c(s)}{2qvs} + \frac{c(s)}{pvs}\frac{r}{1-r}$, $w_3 = \frac{1}{2} + \frac{c(s)}{2qvs} + \frac{c(s)}{pvs}\frac{1-r}{r}$.

As shown above, $EU^A(s,w|0,0) = EU^A(s,w|1,0)$, which implies that there is no discontinuity at w_1 if the equilibrium selected in the multiplicity region is $\sigma^A = 1, \sigma^B = 0$. If the equilibrium selected in the region of multiplicity is $\sigma^A = 0, \sigma^B = 1$, then at w_1 we have the same discontinuity as in (41). If the mixing equilibrium is selected in the region of multiplicity, then at w_1 we have $\sigma^A = 1, \sigma^B = 0$, and therefore $EU^A(s,w_1|(0,0)) = EU^A(s,w_1|(\sigma^A,\sigma^B))$. Hence, there is no discontinuity at w_1.

At w_2, if the equilibrium selected in the multiplicity region is $\sigma^A = 0, \sigma^B = 1$, then there is no discontinuity, as the equilibrium in (w_2, w_3) is also $\sigma^A = 0, \sigma^B = 1$. If the equilibrium selected in the multiplicity region is $\sigma^A = 1, \sigma^B = 0$, then

$$\lim_{w\uparrow w_2} EU^A(s,w|(1,0)) = \frac{sv}{2} + \frac{c(s)}{2q} + \frac{c(s)}{p}\frac{2r-1}{1-r},$$

$$\lim_{w\downarrow w_2} EU^A(s,w|(0,1)) = \frac{sv}{2} + \frac{c(s)}{2q} - \frac{c(s)}{p(1-r)} - c(s)\frac{2(1-2r)}{pr+q(1-r)}.$$

Then,

$$\lim_{w\uparrow w_2} EU^A(s,w) - \lim_{w\downarrow w_2} EU^A(s,w) = c(s)\frac{2}{p}\frac{p(1-r)+qr}{pr+q(1-r)} > 0.$$

Hence, there is a jump down at w_2.

If the mixing equilibrium is selected in the multiplicity region, then at w_2, $\sigma^A = 1, \sigma^B = \frac{r}{1-r}$.

$$\lim_{w\uparrow w_2} EU^A(s,w|(\sigma^A,\sigma^B)) = \frac{sv}{2} + \frac{c(s)}{2q} - \frac{c(s)}{2p(1-r)}$$

Then,

$$\lim_{w\uparrow w_2} EU^A(s,w) - \lim_{w\downarrow w_2} EU^A(s,w) = \frac{c(s)}{2p(1-r)} + c(s)\frac{2(1-2r)}{pr+q(1-r)} > 0.$$

Hence, there is also a jump down at w_2 under mixing.

Finally, at w_3,

$$\lim_{w\uparrow w_3} EU^A(s,w) = \frac{sv}{2} + \frac{c(s)}{2q} - \frac{c(s)}{pr} = \lim_{w\downarrow w_3} EU^A(s,w)$$

Thus, there is no jump at w_3 if $r < 0.5$ ■

Auxiliary Lemma 6 *It is never the case that* $\frac{\partial EU^A(s,w|(0,1))}{\partial w} > 0 > \frac{\partial EU^A(s,w|(1,1))}{\partial w}$.

Proof.

From (38) and (39), given that $\frac{1-r}{r} > \frac{1-2r}{r(1-r)}$, we have

$$\begin{cases} 0 < \frac{\partial EU^A(s,w|(0,1))}{\partial w}, 0 < \frac{\partial EU^A(s,w|(1,1))}{\partial w} & \text{if } p > q\frac{1-r}{r} \\ \frac{\partial EU^A(s,w|(0,1))}{\partial w} < 0 < \frac{\partial EU^A(s,w|(1,1))}{\partial w} & \text{if } q\frac{1-2r}{r(1-r)} < p < q\frac{1-r}{r} \\ \frac{\partial EU^A(s,w|(0,1))}{\partial w} < 0, \frac{\partial EU^A(s,w|(1,1))}{\partial w} < 0 & \text{if } p < q\frac{1-2r}{r(1-r)} \end{cases}. \tag{42}$$

■

Consider the problem for Politician A of choosing w for a given s. Denote this value $w^*(s)$.

Auxiliary Lemma 7 *If $r > 0.5$, then either $w^*(s) = w_3(s) < 1$ or $w^*(s) = 1$.*

Proof. By Auxiliary Lemmas 1, 2 and 4, the solution $w^*(s) \in \{w_1(s), w_2(s), w_3(s), 1\}$. If $r > 0.5$, then $w^*(s) \notin (w_1, w_2)$ if $w_2(s) \leq 1$, given that the function is monotone in between these bounds, by Lemmas 2-4. By Auxiliary Lemma 5, there is either a jump down or continuity at w_1, followed by a jump up or continuity at w_2. Hence, the equilibrium selection in the multiplicity region is irrelevant for the value of $w^*(s)$. Auxiliary Lemma

2 shows that $EU^A(s,w)$ is increasing for $w \leq w_1(s)$ and for $w \in [w_2(s), w_3(s)]$. Lemma 6 implies that $\frac{\partial EU^A(s,w|(1,1))}{\partial w} > 0$, since $q\frac{1-2r}{r(1-r)} < 0$. Therefore, the expected utility is increasing for all $w \notin (w_1, w_2)$. By Lemma 5, the only discontinuity for $w \notin [w_1, w_2]$ is at $w_3(s)$, where the function jumps down. Therefore, the maximum satisfies $w^*(s) \in \{w_3(s), 1\}$. ■

Auxiliary Lemma 8 *For $r < 0.5$,*

- *if in the multiplicity region the equilibrium selected is $\sigma^A = 0, \sigma^B = 1$ or the mixing equilibrium, then $w^*(s) = w_1(s) < 1$ or $w^*(s) = 1$;*
- *otherwise, $w^*(s) = w_2(s) < 1$ or $w^*(s) = 1$.*

Proof. By Auxiliary Lemmas 1, 2 and 4, the solution $w^*(s) \in \{w_1(s), w_2(s), w_3(s), 1\}$. But Auxiliary Lemma 6 implies that the solution cannot be $w_3(s)$. If the selected equilibrium is $\sigma^A = 0, \sigma^B = 1$ in the multiplicity region, then the expected utility function has the same expression for $w \in [w_1(s), w_3(s)]$ and is monotone in this interval, hence the solution cannot be at $w_2(s) \in [w_1(s), w_3(s)]$. By Lemma 5, there is jump down at $w_1(s)$ and the expected utility is otherwise continuous. Therefore, $w^*(s) \in \{w_1(s), 1\}$.

If in the multiplicity region the equilibrium selected is the mixing equilibrium, then by Lemma 5, the only discontinuity is at $w_2(s)$, where the expected utility jumps down. By Lemma 4, the expected utility is decreasing in the mixing region. Hence, $w_2(s)$ cannot be the solution. Therefore, $w^*(s) \in \{w_1(s), 1\}$.

If in the multiplicity region the equilibrium selected is $\sigma^A = 1, \sigma^B = 0$, then Lemma 2, the expected utility is increasing for all $w \leq w_2(s)$. By Lemma 5, the only discontinuity is at $w_2(s)$, where the expected utility jumps down. Hence, $w^*(s) \in \{w_2(s), 1\}$. ■

Optimal s

Given $w^*(s)$, we can now move to the selection of s. Notice that given $w = 1$, the expression for $EU^A(s|w = 1)$ implied by (17) is strictly concave in s for any $\sigma^A, \sigma^B \in \{0, 1\}$. Moreover, it is either concave or convex and increasing in s for the σ^A, σ^B given in the mixing equilibrium.

Part 1: Solution when $r > 0.5$. Consider first the case when the equilibrium selected in the multiplicity region is $(\sigma^A, \sigma^B) = (1, 0)$, such that in the revision equilibrium we have $\sigma^B = 0$. Then by Auxiliary Lemma 7, $w^*(s) \in \{w_3(s), 1\}$. We have $w_3 \leq 1$ iff $\frac{c(s)}{s} < vq\frac{p(1-r)}{p(1-r)+2qr}$. Additionally, if $w_3 \leq 1$, then the equilibrium at $w = 1$ has $\sigma^B = 1$; else, it has $\sigma^B = 0$. Therefore, if $\frac{c(s)}{s} \geq vq\frac{p(1-r)}{p(1-r)+2qr}$, we have $\sigma^B = 0$, and the maximization problem over s becomes

$$\max sv - \frac{c(s)}{p}. \tag{43}$$

This leads to

$$c'(s^*) = \max\{vp, c'(\overline{s})\}, \tag{44}$$

where

$$\frac{c(\overline{s})}{\overline{s}} = vq\frac{p(1-r)}{p(1-r)+2qr}. \tag{45}$$

If $s \leq \overline{s}$, then $w_3 \leq 1$, and

$$EU^A(s|w_3,(1,0)) = \frac{vs}{2} + c(s)\left(\frac{2r-1}{p(1-r)} + \frac{1}{2q}\right), \tag{46}$$

which is strictly convex and increasing in s. Then, the optimal s is $\overline{s}$. Thus,

$$EU^A(\overline{s}) = \overline{s}\cdot v\cdot\left(1 - \frac{q(1-r)}{p(1-r)+2qr}\right) \tag{47}$$

At $w=1$, with $w_3<1$ such that $\sigma^B=1$, we have $EU^A(s|w=1,(1,1)) > EU^A(s|w=w_3,(1,0))$ if

$$\frac{c(s)}{s} \leq \frac{q(2r-1)+p(1-r)r)}{(p(1-r)+q)2q(1-r)+(2(2r-1)q+p(1-r))(q(1-r)^2+qr^2+pr(1-r))}.$$

Notice that

$$\begin{aligned} EU^A(s|w=1,(1,1)) &\leq sv\frac{q(1-r)^2+(q(2r-1)+p(1-r)r)}{q(1-2r(1-r))+pr(1-r)} \\ &\leq \overline{s}v\frac{p(1-r)+2qr-q(1-r)}{p(1-r)+2qr} = EU^A(\overline{s}|w=1,(1,0)). \end{aligned}$$

Therefore, the maximum utility value reached in the region of s values where $w_3<1$ is below the utility reached when $s=\overline{s}$. Hence, the Politician $A's$ optimal choices are $w^*=1$, and s^* given in (44):

$$c'(s^*) = \begin{cases} vp \text{ if } p \geq \overline{q}(\varepsilon,q,r|r>0.5) \\ c'(\overline{s}) \text{ if } p < \overline{q}(\varepsilon,q,r|r>0.5) \end{cases}, \tag{48}$$

where

$$\overline{q}(\varepsilon,q,r|r>0.5) = q\cdot\left(\varepsilon(\overline{s}) - \frac{2r}{1-r}\right). \tag{49}$$

The equilibrium revision strategies are $(0,0)$ if $w_1 \geq 1$, that is, if $s^* > \overline{s}$ and $qv \leq c(s^*)/s^*$. The revision equilibrium is $(\sigma^A,\sigma^B)=(1,0)$ otherwise.

Next, consider the case when the equilibrium selected in the multiplicity region is not $(\sigma^A,\sigma^B)=(1,0)$. The analysis is as above if $w_3(s)\leq 1$. If $w_1(s)\leq 1 < w_2(s)$, i.e., if $vq\frac{pr}{pr+2q(1-r)} < \frac{c(s)}{s} \leq vq$, then the revision equilibrium at $w=1$ does not have $\sigma^B=0$. In this case, given Auxiliary Lemma 5, the solution $w^*(s)$ is $w_1(s)$. Yet, note that $EU^A(s|w=1,(0,1)) \geq EU^A(s|w=w_1,\sigma^B\neq 0)$. Thus, the solution s^* may be in the interval $vq\frac{pr}{pr+2q(1-r)} < \frac{c(s)}{s} \leq vq$ only if at the value s^p for which $c'(s^p) = vp$ we have $\frac{c(s^p)}{s^p} \in (vq\frac{pr}{pr+2q(1-r)}, vq)$. Then s is chosen to maximize

$$\max_{qv\frac{pr}{pr+2q(1-r)} < \frac{c(s)}{s} < qv} \frac{sv}{2} + \frac{c(s)}{2q} - \frac{c(s)}{p}. \tag{50}$$

If $p > 2q$, then the expected utility is increasing in s and the solution is $s = \tilde{s} > \overline{s}$ such that $\frac{c(\tilde{s})}{\tilde{s}} = vq$, given $sc'(s) \geq c(s)$.

If $p < 2q$, then the optimal s, denote it s^i, is such that

$$c'(s^i) = vp\frac{q}{2q-p}, \tag{51}$$

and

$$c'(s^{w2}) \leq c'(s^i) \leq c'(s^{w1}), \tag{52}$$

where

$$\frac{c(s^{w2})}{s^{w2}} = vq\frac{pr}{pr+2q(1-r)},$$
$$\frac{c(s^{w1})}{s^{w1}} = vq.$$

Finally, the solution s^* is as given by s^i derived in (51) if the global maximum is in this interval, i.e., if

$$EU^A(s^i|w = w_1, \sigma^B \neq 0) \geq EU^A\left(s|\frac{c(s)}{s} = vq\frac{pr}{pr+2q(1-r)}, w = 1, (0,1)\right), \tag{53}$$

$$EU^A(s^i|w = w_1, \sigma^B \neq 0) \geq EU^A\left(s|\frac{c(s)}{s} = vq, w = 1, (0,1)\right). \tag{54}$$

To sum up, if in the mixing region an equilibrium with $\sigma^B \neq 0$ is chosen, then the solution differs from the case when the equilibrium selection is $\sigma^B = 0$ only if $p < 2q$ and conditions (52), (53) and (54) are satisfied; in this case, $w^* = w_1(s^*)$ and $s^* = s^i$ given implicitly in (51).

Part 2: Solution when $r < 0.5$ In this case, by Auxiliary Lemma 8, $w^*(s) \in \{w_1(s), w_2(s), 1\}$. Consider first the case where the equilibrium selected in the multiplicity region is not $(\sigma^A, \sigma^B) = (1, 0)$, such that $w^*(s) \in \{w_1(s), 1\}$.

Part 2A. When $p < q\frac{1-2r}{r(1-r)}$: only solution is $w_1(s)$

By Auxiliary Lemma 6, $w_1(s)$ is the only solution if $p < q\frac{1-2r}{r(1-r)}$.

Part 2A(i): $s \geq \bar{s}_1$ **where** $\frac{c(\bar{s}_1)}{\bar{s}_1} = qv$. In this case, $w_1(s) \geq 1$ and the expected utility at w_1 is

$$EU^A(s|w_1, (0,0)) = vs - \frac{c(s)}{p}. \tag{55}$$

The solution is

$$c'(s) = \max\left\{vp, c'(\bar{s}_1)\right\}.$$

Part 2A(ii): $s < \bar{s}_1$. Here, $w_1(s) = \frac{1}{2} + \frac{c(s)}{s}\frac{1}{2qv}$ is interior, and the expected utility at w_1 is

$$EU^A(s|w_1, (0,0)) = \frac{vs}{2} - c(s)\left(\frac{1}{p} - \frac{1}{2q}\right). \tag{56}$$

The solution in this case is

$$c'(s) = \begin{cases} c'(\bar{s}_1) \text{ if } p \geq 2q \\ \min\left\{\frac{vqp}{2q-p}, c'(\bar{s}_1)\right\} \text{ if } p \in [q, 2q] \\ \frac{vqp}{2q-p} \text{ if } p \leq q \end{cases} \tag{57}$$

Therefore, the solution when $p < q\frac{1-2r}{r(1-r)}$ is

$$c'(s^*) = \begin{cases} \frac{vqp}{2q-p} \text{ if } p \leq \underline{q}(\varepsilon, q, r|r < 0.5), \\ c'(\bar{s}_1) \text{ if } \underline{q}(\varepsilon, q, r|r < 0.5) < p < \bar{q}(\varepsilon, q, r|r < 0.5), \\ vp\text{if } p \geq \bar{q}(\varepsilon, q, r|r < 0.5), \end{cases} \tag{58}$$

where

$$\bar{q}(\varepsilon, q, r|r < 0.5) = q \cdot \frac{c'(\bar{s}_1)\bar{s}_1}{c(\bar{s}_1)}, \tag{59}$$

$$\underline{q}(\varepsilon, q, r|r < 0.5) = 2q \cdot \frac{1}{1 + \frac{c(\bar{s}_1)}{c'(\bar{s}_1)\bar{s}_1}} \tag{60}$$

Part 2B. When $q\frac{1-2r}{r(1-r)} < p$: solution is either $w_1 \leq 1$ or $w = 1$.

In this case, it is possible that there is a corner solution at $w = 1$.

Part 2B (i). $s \leq \bar{s}_3$, **where** $\frac{c(\bar{s}_3)}{\bar{s}_3} = vq\frac{pr}{pr+2q(1-r)}$**.**

The threshold w_3 is interior if $s \leq \bar{s}_3$. Then, the revision equilibrium at $w = 1$ is $(\sigma^A, \sigma^B) = (1, 1)$. This means that

$$EU^A(s|w = 1, (1,1)) = vs\frac{r(p(1-r) + qr)}{r(p(1-r) + qr) + q(1-r)^2} - \frac{c(s)}{p}\frac{p(1-r) + q}{r(p(1-r) + qr) + q(1-r)^2}, \tag{61}$$

The corner $w = 1$ is optimal if $EU^A(s|w = 1, (1, 1)) - EU^A(s|w_1, (0, 0)) \geq 0$, which is equivalent to

$$\frac{c(s)}{s} \leq \frac{c(\overline{s}_{11})}{\overline{s}_{11}} \equiv vqp \cdot \frac{pr(1-r) - q(1-2r)}{(1-r)r(p-2q)^2 + pq(3-2r)}, \tag{62}$$

where it can be verified that $\overline{s}_{11} < \overline{s}_3$. Else, if $s > \overline{s}_{11}$, then $w_1(s)$ is optimal.

This implies that in the region $s[0, \overline{s}_{11}]$, the optimal s is

$$c'(s(1,1)) = \min\left\{vp \cdot \frac{r(p(1-r)+qr)}{p(1-r)+q}, c'(\overline{s}_{11})\right\},$$

whereas in the region $s > \overline{s}_{11}$, the solution is

$$c'(s^*) = \begin{cases} \frac{vqp}{2q-p} \text{ if } \max\{\frac{vqp}{2q-p}, vp\} \leq c'(\overline{s}_1) \text{ and } p < 2q \\ c'(\overline{s}_1) \text{ if } vp < c'(\overline{s}_1) < \frac{vqp}{2q-p} \text{ and } q < p < 2q \text{ or } vp < c'(\overline{s}_1) \text{ and } p \geq 2q \\ vp \text{ if } c'(\overline{s}_1) \leq vp. \end{cases} \tag{63}$$

We will next examine each of these four cases. In the first case, if if $c'(s^*) = \frac{vqp}{2q-p}$, then note that

$$\frac{p(1-r)+q}{r(p(1-r)+qr)+q(1-r)^2} > \frac{2q-p}{2q}, \tag{64}$$

which means that

$$\frac{\partial^2 EU^A(s|w=1,(1,1))}{\partial s^2} < \frac{\partial^2 EU^A(s|w=w_1,(0,0))}{\partial s^2}, \tag{65}$$

and therefore a sufficient condition for the global maximum to satisfy $s^* \geq \overline{s}_{11}$ is that

$$-\frac{\partial EU^A(\overline{s}_{11}|w=1,(1,1))}{\partial s} < \frac{\partial EU^A(\overline{s}_{11}|w=w_1,(0,0))}{\partial s},$$

which reduces to

$$c'(\overline{s}_{11}) \leq vpq\frac{(3r(1-r)p + q(1-2r+4r^2))}{pq(1-2r)+4q^2-(p-2q)^2 r(1-r)}. \tag{66}$$

Given (62), the above implies an upper bound on the elasticity of $c(s)$ of

$$\frac{s \cdot c'(s)}{c(s)} \leq \min_{p\in(q\frac{1-2r}{r(1-r)},2q), q\in[0,1], r\in[0,0.5]} \left\{\frac{r(1-r)(p-2q)^2 + pq(3-2r)}{p(1-r)r - q(1-2r)} \cdot \frac{(3r(1-r)p + q(1-2r+4r^2))}{pq(1-2r)+4q^2-(p-2q)^2 r(1-r)}\right\}.$$

This bound is decreasing in r, which means that the problem can be reduced to

$$\frac{s \cdot c'(s)}{c(s)} \leq \min_{p \in (0,2q), q \in [0,1]} \left\{ \frac{(p+2q)(3p+4q)}{p(6q-p)} \right\}. \tag{67}$$

Using numerical minimization methods, we can show that this bound for the elasticity is at least 4.19. Therefore, a sufficient condition for the solution $s*$ to satisfy $s^* \geq \overline{s}_{11}$ is

$$\frac{c'(s) \cdot s}{c(s)} \leq 4 \leq \min_{p \in [0,2q], q \in [0,1]} \left\{ \frac{(p+2q)(3p+4q)}{p(6q-p)} \right\}. \tag{68}$$

Next, consider the second case, where $c'(\overline{s}_1) = \frac{vqp}{2q-p}$ and $q < p$. Notice that at $q = p$, we have $s^*_{00} = \arg\max EU^A(s|w_1, (0,0))$ that satisfies $c'(s^*_{00}) = vq \leq c'(\overline{s}_1)$. Thus, the maximum value $EU^A(s|w_1, (0,0))$ is achieved at $s < \overline{s}_1$ for $q = p$. Let s^*_{11} denote the argmax for $\arg\max EU^A(s|w = 1, (1,1))$ and let $r = 1/2$, in order to maximize $EU^A(s^*_{11}|w = 1, (1,1))$. The effect of decreasing q at $s = \overline{s}_{11}$ is

$$-\frac{\partial^2 EU^A(s|w = 1, (1,1))}{\partial s \partial q} = \frac{vp}{(p+2q)^2}, \tag{69}$$

$$-\frac{\partial^2 EU^A(s|w_1, (0,0))}{\partial s \partial q} = \frac{c'(s^*_{00})}{2q^2}, \tag{70}$$

$$-\frac{\partial^3 EU^A(s|w = 1, (1,1))}{\partial s \partial q} = 0, \tag{71}$$

$$-\frac{\partial^3 EU^A(s|w_1, (0,0))}{\partial s^2 \partial q} = \frac{c''(s^*_{00})}{2q^2}. \tag{72}$$

This implies

$$-\left(\frac{\partial^2 EU^A(s|w = w_1, (0,0))}{\partial s \partial q} + \frac{\partial^2 EU^A(s|w = 1, (1,1))}{\partial s \partial q} \right) > 0, \tag{73}$$

while the rate of decrease in the slope of $EU^A(s)$ decreases relatively more for $EU^A(s|w_1, (0,0))$ compared to $EU^A(s|w = 1, (1,1))$. Then, (73) together with (71) and (72) along with condition (68) imply that

$$\max_{s \leq \overline{s}_1} EU^A(s|w_1, (0,0)) > \max_{s \leq \overline{s}_1} EU^A(s|w = 1, (1,1)) \text{for } q \in [0,p]. \tag{74}$$

In the third case, if $c'(s) = vp$, then notice that

$$EU^A(s|w = 1, (0,0)) - EU^A(s|w = 1, (1,1)) > 0, \tag{75}$$

and
$$\left.\frac{\partial EU^A(s|w=1,(0,0))}{\partial s}\right|_{s=\overline{s}_1} > 0 \implies \left.\frac{\partial EU^A(s|w=w_1,(0,0))}{\partial s}\right|_{s=\overline{s}_1} \geq 0.$$

Then, the $\arg\max_{s\geq\overline{s}_1} EU^A(s|w=1,(0,0))$ is the global maximum.

In sum, under condition (68), the solution is

$$\begin{cases} c'(s^*) = \frac{vqp}{2q-p}, w^* = w_1 < 1 & \text{if } p \leq \underline{q}, \\ c'(s^*) = c'(\overline{s}_1), w^* = 1 & \text{if } \underline{q} < p < \overline{q}, \\ c'(s^*) = vp, w^* = 1 & \text{if } p \geq \overline{q}, \end{cases} \tag{76}$$

where $\overline{q}$ and $\underline{q}$ are given in (59).

Other equilibrium selections under multiplicity. Consider next the case where the equilibrium selected in the multiplicity region is $(\sigma^A, \sigma^B) = (1,0)$. In this case, $w^* \in \{w_2(s), 1\}$. The analysis in Part 2A carries over under the change from w_1 to w_2 and therefore the solution for s^* and w^* becomes

$$\begin{cases} c'(s^*) = \frac{vqp(1-r)}{2q(1-2r)-p(1-r)}, w^* = w_1 < 1 & \text{if } p \leq \underline{q}^{alt}, \\ c'(s^*) = c'(\overline{s}_2), w^* = 1 & \text{if } \underline{q}^{alt} < p < \overline{q}^{alt}, \\ c'(s^*) = vp, w^* = 1 & \text{if } p \geq \overline{q}^{alt}, \end{cases} \tag{77}$$

where

$$\overline{s}_2 = vq\frac{p(1-r)}{p(1-r)+2qr} \tag{78}$$

$$\underline{q}^{alt} = 2q\frac{(1-2r)\frac{\overline{s}_2 c'(\overline{s}_2)}{c(\overline{s}_2)} - r}{(1-r)\left(1+\frac{\overline{s}_2 c'(\overline{s}_2)}{c(\overline{s}_2)}\right)} \tag{79}$$

$$\overline{q}^{alt} = q\left(\frac{\overline{s}_2 c'(\overline{s}_2)}{c(\overline{s}_2)} - \frac{2r}{1-r}\right) \tag{80}$$

For Part 2B, notice that $EU^A(s|w_2,(0,0)) \geq EU^A(s|w_1,(0,0))$, which means that condition (68) is sufficient to ensure that the solution is not $w=1$ and $(\sigma^A,\sigma^B)=(1,1)$. The solution therefore is as in (76), replacing $\overline{q}$ and $\underline{q}$ by $\overline{q}^{alt}$ and $\underline{q}^{alt}$, respectively, and $\frac{c(\overline{s}_{11})}{\overline{s}_{11}}$ by $\frac{c(\overline{s}_{11}^{alt})}{\overline{s}_{11}^{alt}}$ with

$$\frac{c(\overline{s}_{11}^{alt})}{\overline{s}_{11}^{alt}} = vqp\cdot\frac{(1-r)(pr(1-r)-q(1-2r))}{(2qr+(1-r)p)((p+4q)r(1-r)+q(3-2r))} \tag{81}$$

Finally, if the equilibrium in the multiplicity region is mixing, then the analysis the same as in parts 2A and 2B above.

Part 3: Over-scaling or under-scaling. The unconstrained scale in the benchmark with no transitions in control: $c'(s^{NT}) = vp$. Hence, given the above derivations, for $r \geq \frac{1}{2}$,

$$s^* \begin{cases} = s^{NT} \text{ if } p \geq \tilde{q} \\ > s^{NT} \text{ if } p < \tilde{q} \end{cases}.$$

Notice that the case $s^* > s^{NT}$ requires $\epsilon(\overline{s}) > 2$.

If $r < \frac{1}{2}$ and the equilibrium selected in the multiplicity region is not $(\sigma^A, \sigma^B) = (1, 0)$, then notice that $vq\frac{q}{2q-p} < vp$ implies $p < q$. Thus,

$$s^* \begin{cases} = s^{NT} \text{ if } p \geq \overline{q} \text{ or } p = q \\ > s^{NT} \text{ if } q < p < \overline{q} \\ < s^{NT} \text{ if } p < q \end{cases}.$$

If $r < \frac{1}{2}$ and the equilibrium selected in the multiplicity region is $(\sigma^A, \sigma^B) = (1, 0)$, then

$$s^* \begin{cases} = s^{NT} \text{ if } p \geq \overline{q}^{alt} \text{ or } p = q \\ > s^{NT} \text{ if } q < p < \overline{q}^{alt} \\ < s^{NT} \text{ if } p < q \end{cases}.$$

A.3 Proof to Corollary 1

Follows from Proposition 2. For $r \geq 1/2$, $c'(s^*) = vp$, which is increasing in p. For $r < 1/2$, the result follows given the expressions for s^* in (76) or (77).

A.4 Proof to Proposition 3

From the proof to Proposition 2, if $r > 1/2$, then for $s^{\max}$ such that $c'(s^{\max}) < 2qp(1 - r)/(p(1-r) + 2qr)$, the optimal solution $w(s)$ for any $s \leq s^{\max}$ is $w(s) = 1$ and the revision equilibrium is $(\sigma^A, \sigma^B) = (1, 1)$.

If $r < 1/2$ and $s^{\max} < \min \overline{s}_{11}, \overline{s}_{11}^{alt}$, then $s^* = s^{\max}$, $w(s) = 1$ and the revision equilibrium is $(\sigma^A, \sigma^B) = (1, 1)$.

Notice that $\overline{s}_{11}$, $\overline{s}_{11}^{alt}$, and vp are all increasing in p. Thus, for every $p^* \in (0, 1)$, let

$$c'(s^M(p^*)) = \begin{cases} \frac{2qp(1-r)}{(p(1-r)+2qr)} & \text{if } r \geq \frac{1}{2} \\ c'(\overline{s}_{11}(p^*)) & \text{if } r < \frac{1}{2} \text{ and } (1,0) \text{ not selected under multiplicity} \\ c'(\overline{s}_{11}^{alt}(p^*)) & \text{if } r < \frac{1}{2} \text{ and } (1,0) \text{ selected under multiplicity} \end{cases} \quad (82)$$

Let $s^{\max} = s^M(p^*)$. Then, for all $p < p^*$, $c'(s^M(p^*)) > \frac{2qp(1-r)}{(p(1-r)+2qr)}$, which implies that the solution for $r > \frac{1}{2}$ is $s^* = qv$ if $c'(s^{\max}) < vp$ or $c'(s^*) = vp$ otherwise. For $r < \frac{1}{2}$,

$s^M(p^*) > \bar{s}_{11}(p)$ and $s^M(p^*) > \bar{s}_{11}^{alt}(p)$, and so the solution s^* is in the equilibrium with $\sigma^B = 0$. This implies no revisions on the equilibrium path.

For all $p \geq p^*$, as shown above, the only possible solution is $w = 1$, $s^* = s^{\max}$ and the revision equilibrium $(\sigma^A, \sigma^B) = (1, 1)$.

A.5 Proof to Corollary 2

Follows from the proof of Proposition 3.

A.6 Proof to Proposition 5

Observe first that because revisions in phase 1 cannot affect payoffs in phase 2, revision strategies are identical to those in the one-phase game. As the result focuses on phase 1 strategies, we omit notation for phases.

Part 1: $r > 1/2$. Following the proof of Proposition 2, distribution and revision strategies given s are as follows:

$$\begin{cases} \sigma^A = 1, \sigma^B = 1, \Delta = 1 & s \leq \hat{s}_1 & (a) \\ \sigma^A = 1, \sigma^B = 0, \Delta = w_3 & s \in (\hat{s}_1, \hat{s}_3] & (b) \\ \sigma^A = 0, \sigma^B = 0, \Delta = 1 & s > \hat{s}_3 & (c) \end{cases}$$

where $w_3 = \frac{1}{2} + \frac{s}{2qv} + \frac{sr}{pv(1-r)}$ and:

$$\begin{aligned} \hat{s}_1 &= \frac{pq(1-r)v[p(1-r)r + q(2r-1)]}{(p(1-r)+2qr)\left[p(1-r)r + q\left(4r^2 - 6r + 3\right)\right]} \\ \hat{s}_3 &= \frac{pq(1-r)v}{p(1-r)+2qr}, \end{aligned}$$

which satisfies $0 < \hat{s}_1 < \hat{s}_3$.

Using $\tilde{U}_2^A$ to denote A's ex ante expected value of a single phase of play when $m = 1$ (13), the corresponding objective for agent A is:

$$\hat{V}^A(s) = \begin{cases} V_a^A(s) = \left(\frac{rv(p(1-r)+qr)}{p(1-r)r+q(2r^2-2r+1)} + \tilde{U}_2^A\right) s - \frac{(q+p(1-r))s^2}{p(p(1-r)r+q(2r^2-2r+1))} & s \leq \hat{s}_1 & (a) \\ V_b^A(s) = (\frac{v}{2} + \tilde{U}_2^A)s + \left(\frac{1}{2q} + \frac{2r-1}{p(1-r)}\right) s^2 & s \in (\hat{s}_1, \hat{s}_3] & (b) \\ V_c^A(s) = (v + \tilde{U}_2^A)s - \frac{s^2}{p} & s > \hat{s}_3 & (c) \end{cases}$$

We note several properties of $\hat{V}^A(s)$ and its components. It is straightforward to verify that $\hat{V}^A(s)$ is continuous, concave in regions (a) and (c), and convex in region (b). Additionally, $V_a^A(0) = V_b^A(0) = V_c^A(0) = 0$. Finally, $\frac{dV_c^A(s)}{ds} > \frac{dV_a^A(s)}{ds}$. Together, these facts imply that $\hat{V}^A(s)$ can be maximized only at 0, $\hat{s}_1$, $\hat{s}_3$, or s_a or s_c, the interior values of s that maximize $V_a^A(s)$ or $V_c^A(s)$, respectively, if they exist.

Taking first order conditions yields the following candidate interior solutions:

$$s_a = \frac{p}{2}\left(r(v+\tilde{U}_2^A) + \frac{q(1-r)[(1-2r)\tilde{U}_2^A - rv]}{p(1-r)+q}\right) \tag{83}$$

$$s_c = \frac{p}{2}(v+\tilde{U}_2^A). \tag{84}$$

We make three observations that narrow the set of possible solutions. First, the region (c) solution is interior (i.e., $s_c > \hat{s}_3$) if:

$$\tilde{U}_2^A > \phi_c \equiv -\frac{v(p(1-r)+2q(2r-1))}{p(1-r)+2qr}. \tag{85}$$

Second, $s_a > 0$ if:

$$\tilde{U}_2^A > \phi_a \equiv -\frac{vr(p(1-r)+qr)}{p(1-r)r+q\left(2r^2-2r+1\right)}. \tag{86}$$

Third, $\hat{V}^A(\hat{s}_1) < \hat{V}^A(\hat{s}_3)$ if:

$$\tilde{U}_2^A > \phi_b \equiv -\frac{v\left(3p^2(1-r)^2r + pq(1-r)(2r(9r-7)-5) - 2q^2(6(3-2r)r^2-11r)+2)\right)}{2(p(1-r)+2qr)\left(p(1-r)r+q(4r^2-6r+3)\right)}. \tag{87}$$

Under the assumed parameter values, $\phi_b < \phi_a < \phi_c$.

We now derive the optimal s for each possible value of $\tilde{U}_2^A$. There are four cases.

(i) $\tilde{U}_2^A > \phi_c$. Because $V_c^A(s) > V_a^A(s)$ for $s > 0$, $s^* = s_c$.

(ii) $\tilde{U}_2^A \in (\phi_a, \phi_c]$. The possible solutions are s_a and $\hat{s}_3$. Solving $V^A(s_a) = V^A(\hat{s}_3)$ for $\tilde{U}_2^A$ produces a unique value ϕ_{ac} of $\tilde{U}_2^A$ such that $\phi_{ac} \in (\phi_a, \phi_c]$ and $s^* = s_a$ if $\tilde{U}_2^A < \phi_{ac}$ and $s^* = \hat{s}_3$ otherwise. (We omit the expression for ϕ_{ac} due to excessive length.)

(iii) $\tilde{U}_2^A \in (\phi_b, \phi_a]$. The possible solutions are 0 and $\hat{s}_3$. Observe that $V_c^A(\hat{s}_3) \geq 0$ only for $\tilde{U}_2^A > \frac{rv(p(1-r)+qr)}{p(1-r)r+q(2r^2-2r+1)}$, but this value of $\tilde{U}_2^A$ is greater than ϕ_a. Thus the optimal solution must be $s^* = 0$.

(iv) $\tilde{U}_2^A \leq \phi_b$. The possible solutions are 0 and $\hat{s}_1$. $\hat{s}_1$ cannot be the solution because $V_a^A(s)$ is decreasing in s for $s > 0$ when $\tilde{U}_2^A \leq \phi_a$; thus the optimal solution must be $s^* = 0$.

Combining cases produces:

$$s^* = \begin{cases} s_c & \text{if } \tilde{U}_2^A > \phi_c \\ \hat{s}_3 & \text{if } \tilde{U}_2^A \in (\phi_{ac}, \phi_c] \\ s_a & \text{if } \tilde{U}_2^A \in (\phi_a, \phi_{ac}] \\ 0 & \text{if } \tilde{U}_2^A \leq \phi_a. \end{cases} \tag{88}$$

Comparing $\tilde{U}_2^A$ with ϕ_a, it is clear that the project will not be cancelled in phase 1 if $p < q$ or $p > 2q = \overline{q}$.

Part 2: $r < 1/2$. Following the proof of Proposition 2, distribution and revision strategies given s are as follows:

$$\begin{cases} \sigma^A = 1, \sigma^B = 1, \Delta = 1 & s \le \check{s}_1 & (a) \\ \sigma^A = 0, \sigma^B = 0, \Delta = w_1 & s \in (\check{s}_1, \check{s}_3] & (b) \\ \sigma^A = 0, \sigma^B = 0, \Delta = 1 & s > \check{s}_3 & (c) \end{cases}$$

where $w_1 = \frac{1}{2} + \frac{s}{2qv}$ and:

$$\begin{aligned} \check{s}_1 &= \frac{pqv\left[p(1-r)r - q(1-2r)\right]}{(p^2+4q^2)(1-r)r + pq(4r^2-6r+3)} \\ \check{s}_3 &= qv, \end{aligned}$$

which satisfies $\check{s}_1 < \check{s}_2$.

The corresponding objective for agent A is:

$$\check{V}^A(s) = \begin{cases} V_a^A(s) = \left(\frac{rv(p(1-r)+qr)}{p(1-r)r+q(2r^2-2r+1)} + \tilde{U}_2^A\right)s - \frac{(p(1-r)+q)s^2}{p[p(1-r)r+q(2r^2-2r+1)]} & s \le \check{s}_1 & (a) \\ \check{V}_b^A(s) = (\frac{v}{2} + \tilde{U}_2^A)s + \left(\frac{1}{2q} - \frac{1}{p}\right)s^2 & s \in (\check{s}_1, \check{s}_3] & (b) \\ V_c^A(s) = (v + \tilde{U}_2^A)s - \frac{s^2}{p} & s > \check{s}_3 & (c) \end{cases}$$

We note several properties of $\check{V}^A(s)$ and its components. In regions (a) and (c), the component functions are identical to those in Part 1 (and thus concave). $\check{V}_b^A(s)$ is convex if $p > 2q$ and concave otherwise. It is straightforward to verify that $\check{V}^A(s)$ is continuous. Additionally, $V_a^A(0) = \check{V}_b^A(0) = V_c^A(0) = 0$. Together, these facts imply that $\check{V}^A(s)$ can be maximized only at 0, $\check{s}_1$, $\check{s}_3$, or s_a, $\check{s}_b$, or s_c, which are the interior values of s that maximize $V_a^A(s)$, $\check{V}_b^A(s)$, or $V_c^A(s)$, respectively, if they exist. Finally, $\check{s}_3$ is positive but $\check{s}_1$ may be negative; solutions in region (a) can exist only if $\check{s}_1 > 0$.

Taking first order conditions yields the following candidate interior solution for region (b), with the interior solutions s_a and s_c for regions (a) and (c) given by (83) and (84), respectively:

$$\check{s}_b = \frac{pq(v + 2\tilde{U}_2^A)}{4q - 2p}. \tag{89}$$

For $\check{s}_b$ to be interior it must be both positive, which holds if $\tilde{U}_2^A > -v/2$, and in the interval $(\check{s}_1, \check{s}_3]$, which occurs if $\tilde{U}_2^A \in \left(\check{\phi}_b^l, \check{\phi}_b^h\right]$, where:

$$\begin{aligned} \check{\phi}_b^l &= \frac{v\left[4q^2(r^2+r-1) - 3p^2(1-r)r - pq(8r^2-6r+1)\right]}{2\left[(4q^2+p^2)(1-r)r + pq\left(4r^2-6r+3\right)\right]} \\ \check{\phi}_b^h &= v\left(\frac{2q}{p} - \frac{3}{2}\right). \end{aligned}$$

We make three observations that narrow the set of possible solutions. First, the region

(c) solution is interior (i.e., $s_c > \check{s}_3$) if:

$$\tilde{U}_2^A > \check{\phi}_c \equiv v\left(\frac{2q}{p} - 1\right). \tag{90}$$

Second, the region (a) solution is interior (i.e., $s_a \in (0, \check{s}_1)$) if $\tilde{U}_2^A > \phi_a$ (i.e., condition (86) from Part 1), and $\tilde{U}_2^A < \check{\phi}_a^h$, where:

$$\check{\phi}_a^h \equiv \frac{v\left[(1-r)r^2\left(p^3(1-r)+4q^3r\right)+p^2qr\left(1-r(5r^2-9r+5)\right)+ q^2(1-2r)\left(2p+2q-pr(4r^2-5r+4)\right)\right]}{\left((1-r)r(2q-p)-q\right)\left[(1-r)r\left(p^2+4q^2\right)+pq\left(4r^2-6r+3\right)\right]}.$$

The condition $s_a < s_a^h$ is equivalent to $\check{s}_1 > 0$.

Third, if either $\check{s}_b$ or s_c are interior, then A prefers them to $\check{s}_3$, which belongs to both regions (b) and (c). Similarly, if either s_a or $\check{s}_b$ are interior, then A prefers them to $\check{s}_1$.

We now derive the optimal s for each possible value of $\tilde{U}_2^A$ and q.

(i) $\tilde{U}_2^A > \check{\phi}_c$. When $\check{\phi}_c$ is interior, the only possible alternative solutions are 0, $\check{s}_1$, and s_a. By the concavity of $V_c^A(s)$, $V_c^A(s_c) > V_c^A(0)$, and $\frac{dV_c^A(s)}{ds} > \frac{dV_a^A(s)}{ds}$ implies that $V_c^A(s_c) > V_a^A(s_a)$ when $s_a > 0$. Finally, straightforward calculation shows that $V_c^A(s_c) > V_a^A(\check{s}_1)$ when $\check{s}_1 > 0$. Thus, $s^* = s_c$.

For subcases (ii)-(v), $p < 2q$, so the objective $\check{V}_b^A(s)$ in region (b) is concave. As $\tilde{U}_2^A < 0$ for agent A when $r < 1/2$, it it is impossible for condition (90) to be satisfied and thus subcase (i) is irrelevant.

(ii) $\tilde{U}_2^A \in (\check{\phi}_b^h, \check{\phi}_c]$. In this subcase, $\check{V}_b^A(s)$ is maximized at some $s > \check{s}_3$ and $\check{s}_b$ is not feasible. If $\check{s}_1 \leq 0$, then the optimal s is the region (b) corner: $s^* = \check{s}_3$.

If $\check{s}_1 > 0$, then $\check{V}_b^A(\check{s}_3) > \check{V}_b^A(\check{s}_1)$. By the concavity of $\check{V}_b^A(s)$, $\check{V}_b^A(\check{s}_3) > \check{V}_b^A(0)$, and so the remaining candidate solutions are $\check{s}_3$ and s_a. Performing the necessary substitutions and solving, $\check{V}_a^A(s_a) > \check{V}_b^A(\check{s}_3)$ iff $\tilde{U}_2^A < \check{\phi}_{a3}$, where:

$$\check{\phi}_{a3} \equiv \frac{v\left[2q^2 - p^2(1-r)r - pq(r^2+2r-2) - 2q\sqrt{2(1-r)(p(1-r)+qr)(p(1-r)+q)}\right]}{p\left[p(1-r)r + q\left(2r^2-2r+1\right)\right]}. \tag{91}$$

It is straightforward to verify that $\check{\phi}_{a3} < \check{\phi}_c$. Thus we have $s^* = \check{s}_3$ if $\tilde{U}_2^A \in (\check{\phi}_{a3}, \check{\phi}_c]$, and $s^* = s_a$ if $\tilde{U}_2^A \in (\check{\phi}_b^h, \check{\phi}_{a3}]$, where the latter interval may be empty.

(iii) $\tilde{U}_2^A \in (\max\{-v/2, \check{\phi}_b^l\}, \check{\phi}_b^h]$. In this subcase, $\check{s}_b$ is a feasible solution, which A obviously prefers to $\check{s}_1$ and $\check{s}_3$. By the concavity of $V_b^A(s)$, $\check{V}_b^A(\check{s}_b) > \check{V}_b^A(0)$, and so the only other possible candidate solution is s_a, if region (a) is non-empty. Thus $\hat{s}_1 \leq 0$ implies that the solution is $\check{s}_b$. Furthermore, $\hat{s}_1 \leq 0$ also implies that $-v/2 > \check{\phi}_b^l$.

If $\hat{s}_1 > 0$, then performing the necessary substitutions and solving, there exists ϕ_{ab} such that $\check{V}_a^A(s_a) > \check{V}_b^A(\check{s}_b)$ if $\tilde{U}_2^A < \phi_{ab}$, where $\phi_{ab} > \max\{-v/2, \check{\phi}_b^l, \check{\phi}_{a3}\}$ and $\phi_{ab} \in [\phi_a, \phi_a^h]$. (We omit the expression for ϕ_{ab} due to excessive length.) Thus we have $s^* = s_a$ if $\tilde{U}_2^A \in (\max\{-v/2, \check{\phi}_b^l\}, \phi_{ab}]$, and $s^* = \check{s}_b$ if $\tilde{U}_2^A \in (\phi_{ab}, \check{\phi}_b^h]$.

(iv) $\tilde{U}_2^A \leq \max\{-v/2, \check{\phi}_b^l\}$. $\check{V}_b^A(s)$ is strictly decreasing for $s \geq 0$, so if $\check{s}_1 \leq 0$ then the solution is $s^* = 0$. If $\check{s}_1 \geq 0$, the only feasible solutions are the set of region (a) solutions, or $\{0, s_a, \check{s}_1\}$. Thus the solution is $s^* = 0$ for $\tilde{U}_2^A \leq \phi_a$, $s^* = s_a$ for $\tilde{U}_2^A \in (\phi_a, \check{\phi}_a^h]$, and $s^* = \check{s}_1$ for $\tilde{U}_2^A > \check{\phi}_a^h$.

Combining cases (i)-(iv) produces the following optimal scales. For $\check{s}_1 \leq 0$:

$$s^* = \begin{cases} \check{s}_3 & \text{if } \tilde{U}_2^A \in (\check{\phi}_b^h, \check{\phi}_c] \\ \check{s}_b & \text{if } \tilde{U}_2^A \in (-v/2, \check{\phi}_b^h] \\ 0 & \text{if } \tilde{U}_2^A \leq -v/2. \end{cases} \tag{92}$$

And for $\check{s}_1 > 0$:

$$s^* = \begin{cases} \check{s}_3 & \text{if } \tilde{U}_2^A \in (\check{\phi}_{a3}, \check{\phi}_c] \\ s_a & \text{if } \tilde{U}_2^A \in (\check{\phi}_b^h, \check{\phi}_{a3}] \\ \check{s}_b & \text{if } \tilde{U}_2^A \in (\phi_{ab}, \check{\phi}_b^h] \\ s_a & \text{if } \tilde{U}_2^A \in (\max\{-v/2, \check{\phi}_b^l\}, \phi_{ab}] \\ \check{s}_1 & \text{if } \tilde{U}_2^A \in (\check{\phi}_a^h, \max\{-v/2, \check{\phi}_b^l\}] \\ s_a & \text{if } \tilde{U}_2^A \in (\phi_a, \check{\phi}_a^h] \\ 0 & \text{if } \tilde{U}_2^A \leq \phi_a. \end{cases} \tag{93}$$

It is straightforward to verify that at most one of the regions for which $s^* = s_a$ is non-empty.

For subcases (v)-(viii), $p > 2q$, so the objective $\check{V}_b^A(s)$ in region (b) is convex and $\check{s}_b$ is not a feasible solution. Thus the optimal s is either in region (b) (with possible solutions 0, $\check{s}_1$, $\check{s}_3$), or in region (a), as described in subcase (iv). We first consider the subcase where $\check{s}_1 \leq 0$, so region (a) is empty.

(v) $\tilde{U}_2^A \leq \check{\phi}_c$ and $\check{s}_1 \leq 0$. The only possible solutions are 0 and $\check{s}_3$. Solving $\check{V}_b^A(\check{s}_3) \geq 0$ for $\tilde{U}_2^A$ produces:

$$s^* = \begin{cases} s_c & \text{if } \tilde{U}_2^A > \check{\phi}_c \\ \check{s}_3 & \text{if } \tilde{U}_2^A \in (v(q-p)/p, \check{\phi}_c] \\ 0 & \text{if } \tilde{U}_2^A \leq v(q-p)/p. \end{cases} \tag{94}$$

For the remaining subcases, $\check{s}_1 > 0$, so region (a) is non-empty.

(vi) $\tilde{U}_2^A \in (\check{\phi}_a^h, \check{\phi}_c]$ and $\check{s}_1 > 0$. The only feasible solutions are $\check{s}_1$ and $\check{s}_3$. Performing the necessary substitutions and solving, there exists $\check{\phi}_{13}$ such that $\check{V}_a^A(\check{s}_1) > \check{V}_b^A(\check{s}_3)$ iff $\tilde{U}_2^A < \check{\phi}_{13}$, where:

$$\check{\phi}_{13} \equiv \frac{v\left[4pq^2(1-2r)^2 - (1-r)r(3p^3 - 8q^3) - p^2q(12r^2 - 14r + 5)\right]}{2p\left[(1-r)r(p^2 + 4q^2) + pq(4r^2 - 6r + 3)\right]}.$$

It is easily verified that $\check{\phi}_{13} < \check{\phi}_c$. Thus $s^* = \check{s}_1$ if $\tilde{U}_2^A \in (\check{\phi}_a^h, \check{\phi}_{13}]$, and $s^* = \check{s}_3$ if $\tilde{U}_2^A \in (\check{\phi}_{13}, \check{\phi}_c]$, where the former interval may be empty.

(vii) $\tilde{U}_2^A \in (\phi_a, \check{\phi}_a^h]$. In this subcase, the interior solution s_a is feasible. Using expression (91), $\check{V}_a^A(s_a) > \check{V}_b^A(\check{s}_3)$ if $\tilde{U}_2^A > \check{\phi}_{a3}$. Thus $s^* = \check{s}_3$ if $\tilde{U}_2^A \in (\check{\phi}_{a3}, \check{\phi}_a^h]$, and $s^* = s_a$ if $\tilde{U}_2^A \in (\phi_a, \check{\phi}_{a3}]$, where either interval may be empty.

(viii) $\tilde{U}_2^A \leq \phi_a$. Analogously to subcase (iv), $\check{V}_a^A(s)$ is strictly decreasing for $s \geq 0$, so $s^* = 0$.

Combining cases (i) and (vi)-(viii) produces:

$$s^* = \begin{cases} s_c & \text{if } \tilde{U}_2^A > \check{\phi}_c \\ \check{s}_3 & \text{if } \tilde{U}_2^A \in (\check{\phi}_{13}, \check{\phi}_c] \\ \check{s}_1 & \text{if } \tilde{U}_2^A \in (\check{\phi}_a^h, \check{\phi}_{13}] \\ \check{s}_3 & \text{if } \tilde{U}_2^A \in (\check{\phi}_{a3}, \check{\phi}_a^h] \\ s_a & \text{if } \tilde{U}_2^A \in (\phi_a, \check{\phi}_{a3}] \\ 0 & \text{if } \tilde{U}_2^A \leq \phi_a. \end{cases} \tag{95}$$

It is straightforward to verify that at most one of the regions for which $s^* = s_a$ is non-empty.

Summarizing the conditions for cancellation, when $\check{s}_1 > 0$ A cancels the project in phase 1 if $\tilde{U}_2^A \leq \phi_a$. When $\check{s}_1 \leq 0$, A cancels when $\tilde{U}_2^A \leq -v/2$ if $p < 2q$ and $\tilde{U}_2^A \leq v(q-p)/p$ if $p > 2q$. As $\tilde{U}_2^A$ is independent of v and negative and decreasing in v_2 when $r < 1/2$, we conclude that A cancels when v_2 is sufficiently large.